Chapter-1 Simple Mechanism

12.12.19

★ Introduction :-

→ A machine is a device which receives energy + transfer it into some useful work.

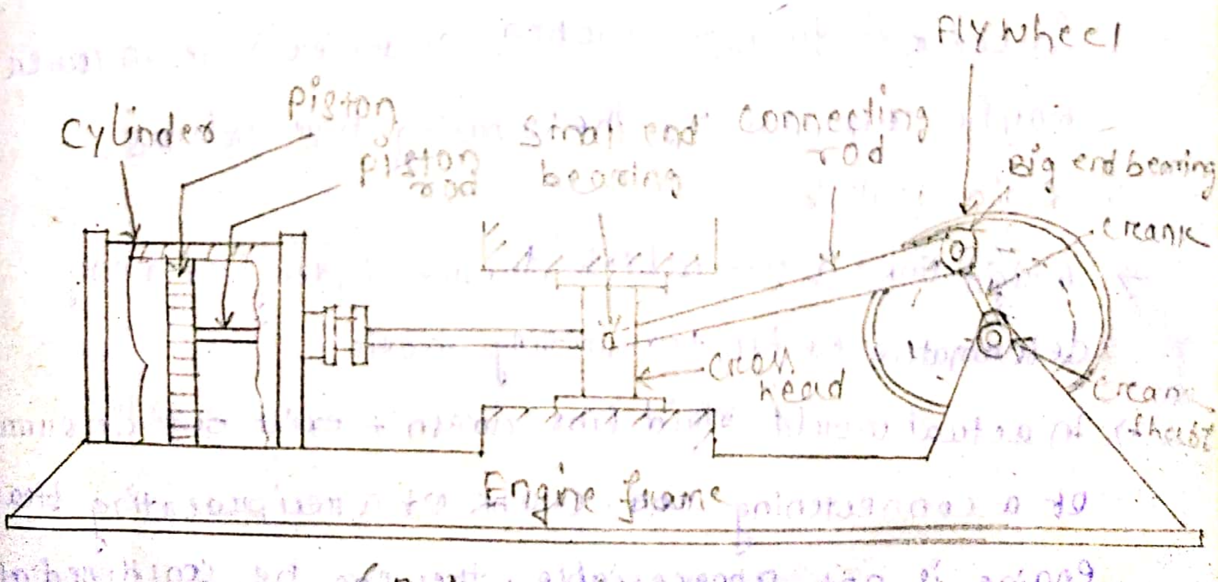
→ A machine consists of no. of parts which are rigidly connected with each other.

→ All the parts or bodies are assembled by making one of the parts as fixed & the relative motion of the other parts is determined w.r.t the fixed part.

Kinematic link or element :-

→ Each part of a machine which moves relative to some other parts is known as kinematic link or simply link or element.

→ A link consists of several parts which are rigidly fastened together so that they don't move relative to one another.



(Reciprocating Steam engine)

Ex:-

In a reciprocating steam engine several parts are assembled by formation of different links. piston, piston rods cross head constitute one link; connecting rod with small & big end bearing constitute second link; crank, crankshaft & fly wheel constitute the third link; cylinder, engine frame & main bearings constitute fourth link.

→ A link or element need not to be a rigid body but it must be a resistant body.

Note:-

- A body is said to be a resistant body if it is capable of transmitting the require forces with negligible deformation.

→ Therefore a link should ~~be a~~ posess the following two characteristics.

a. It should be have relative motion.

b. It must be a resistant body.

14.12.19 Types of links :-

In order to transmit motion the driver & the follower may be connected by the following types of links.

1. Rigid link:-

→ A rigid link is one which doesn't undergoes ~~&~~ any deformation while transmitting motion.

→ In actual world rigid link doesn't exist but deformation of a connectening rod, crank of a reciprocating steam engine is not appreceiable, they can be considered as rigid link.

2. Flexible link :-

→ A flexible link is one which is partly deformed in a manner not to affect the transmission of motion.

→ Ex:- Belts, ropes, chains, wires are flexible link & & transmit tensile forces only.

3. Fluid link :-

→ A fluid link is one which is formed by having a fluid in receptacle /container and the motion is transmitted through by the fluid by pressure or compression only.

→ Ex:- hydraulic presses, jacks & brakes.

Structure :-

→ It is an assemblage of a number of resistant bodies (known as members) having no relative motion betn them & meant for carrying loads.

→ Ex:- railway bridge, a roof truss, machine frames etc.

Difference betn machine & Structure :-

The following differences between a machine & a structure are important.

1. The parts of a machine move relative to one another, whereas the members of a structure donot move relative to one another.

II. A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work.

iii. The links of a machine may transmit both power and motion, while the member of a structure transmit forces only.

19.12.19.

Kinematic pair :-

-> The two links ~~or~~ elements of a machine when in contact with each other then they are said to form a pair.

-> If the relative motion bet^n them is completely or successfully constrained then, the pair is known as Kinematic pair.

Types of constrained motions :-

Following are the three types of constrained motions.

1. Completely constrained motion :-

When the motion between a pair is limited to a definate dir^n only irrespective of the direction of the force applied then the motion is said to be completely constrained motion.

Ex :- 1: piston & cylinder in a steam engine forms a pair the dir^n of a piston is limited to a definate dir^n only i.e it will only reciprocate.

Ex-2 :- The motion of a square bar in a square hole, the motion of a shaft with collar at each end in a circular hole are the example of completely constrained motion.

2. In completely constrained motion :-

when the motion betn the pair can take place in morethan one direction then the motion is called in completly constrained motion.

Ex:- A Circular shaft in a circular hole is an example of incompletely constrained motion as it may either rotate or slide in a hole.

3. Successfully Constrained motion :-

when the motion betn the elements forming a pair is such that the constrained motion is not completed by it self but by some other means, then the motion is said to be successfully constrained motion.

Ex:- Consider a shaft in a foot step bearing. The shaft may rotate in the bearing or it may move upward. this is a case of incomplete constrained motion but if a load is placed on the shaft to prevent the axial upward move-ment of the shaft then the motion of the pair is said to be successfully constrained motion.

Classification of Kinematic pair :-

The kinematic pair may be classified according to the following Consideration.

According to the type of relative motion betn the elements :-

The kinematic pair according to the type of relative motion betn the elements may be classified as follows.

a. Slidding pair :-

When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as sliding pair.

Ex :- Tailstock on a lathe machine is an example of Slidding pair.

b. Turning pair :-

When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair.

Ex :- A shaft with collar at both ends fitted into a circular hole, lathe spindle supported on head stock, cycle wheels turning over the axels are the example of turning pair.

c. Rolling pair :-

When the two elements of a pair are connected in a such a way that one rolls over another fixed link, then the pair is known as rolling pair.

Ex :- ball & roller bearings.

d. Screw pair :-

When two elements of a pair are connected in such a way that, one element can turn about the other by screw thread then the pair is known as screw pair.

Ex:- Lead screw of a lathe, and nut & bolt are the example of screw pair.

e. Spherical pair :-

when the two elements of a pair are connected in such a way that, one element with spherical shape turns or swivels about the other fixed elements then the pair is known as spherical pair.

Ex:- ball & socket joint, attachment of a car mirror are the examples of spherical pairs.

According to the type of contact betn the elements;

According to the type of contacts betn the elements, kinematic pair can be classified into two types :-

a. Lower pair :-

→ when the two elements of a pair have a surface contact when relative motion takes place & the surface of one elements slides over the surface of another, then the pair is known as lower pair.

→ Sliding pair, turning pair & screw pairs are lower pairs.

b. Higher pair :-

→ when the two elements of a pair have a line contact or point contact when relative motion takes place & the motion betn the two elements is partly turning & partly sliding then the pair is known as higher pair.

(rolling pair)

→ Belt & rope drives, ball & roller bearing, cam & follower are example of higher pairs.

According to the type of closure betn the elements:-

According to the type of closure, the kinematic pair can be classified into two types.

a. Self closed pair :-

→ when the two elements of a pair are connected together mechanically in such a way that only require kind of relative motion occurs then the it is known as self closed pair.

→ The lower pairs are self closed pairs.

b. Force closed pair :-

→ when the two elements of a pair are not connected mechanically but are kept in contact by the external forces then the pair is said to be force closed pair.

→ Cam & follower is an example of force closed pairs.

Kinematic chain :-

→ when kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definate motion.

i.e. Completely or successfully constrained motion, then it is called as kinematic chain.

→ In other words the kinematic chain may be define as a combn of kinematic pairs joined in such a way that each link forms a part of two pairs & the relative motion betn the links is completely or successfully constrained.

→ Ex:- Crankshaft of the engine forms ~~a pair or~~ kinematic pair with the bearings, connecting rod with crank form a second kinematic pair, piston with connecting rods forms a third pair & piston with cylinder forms a fourth pair. Therefore the total comb^n of these pairs forms a kinematic chain.

→ If each link is assumed to form two pairs with two adjacent links then the relation bet^n the no. of pairs (p) & no. of links (l) may be expressed in the form of an equation.

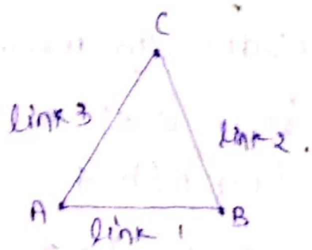
$$l = 2p - 4 \quad -(1)$$

Another relation bet^n the no. of links (l) & no. of joints (j) is given by the equation,

$$j = \frac{3}{2}l - 2 \quad -(2)$$

The equation (1) & (2) are applicable in lower pairs only of a kinematic chain but it may also consider as higher pairs such that each higher pair may be taken as equivalent to two lower pairs.

Cases :- 1



consider the arrangements of three links AB, BC, CA with joints A, B & C.

no. of joints (J) = 3

no. of links (l) = 3

no. of pairs (P) = 3

from eq^n (1)

$l = 2p - 4$

$3 = 2 \times 3 - 4$

$3 = 2$

from L.H.S. > R.H.S.

from eq^n (2)

$J = \frac{3}{2} l - 2$

$3 = \frac{3}{2} \times 3 - 2$

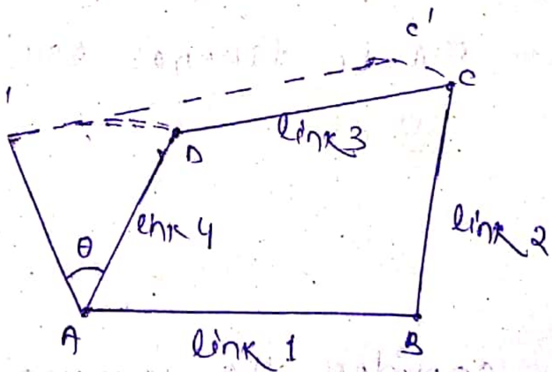
$3 = 2.5$

L.H.S. > R.H.S.

Since the arrangment of three links doesn't satisfy the eq^n of (11) and the ~~left hand s~~ LHS is greater than the RHS therefore it is not a Kinematic chain & hence no relative motion is possible. Such type of chain is called a locked chain and a forms a rigid structure which is used in bridges.

Case-2 :-

Consider an arrangment of four links AB, BC, CD, DA



link (l) = 4

joint (j) = 4

pair (p) = 4

from eq^n (1), $l = 2p - 4$

$4 = 2 \times 4 - 4$

$4 = 4$

L.H.S = R.H.S

From eq^n-(2), $J = \frac{3}{2}l - 2$

$4 = \frac{3}{2} \times 4 - 2$

$4 = 6 - 2$

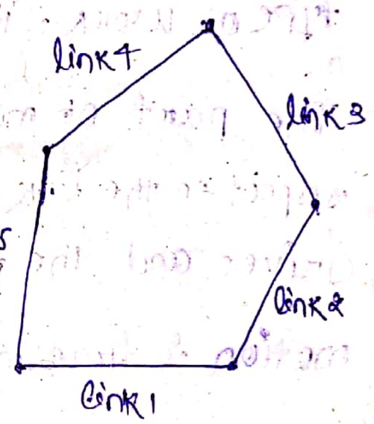
$4 = 4$

L.H.S. = R.H.S.

Since the arrangment of four links satisfy the equations (1) & (2), therefore it is a kinematic chain of one degree of freedom.

A single link AD is sufficient to define the position of all other links, it is then called as kinematic chain of one degree of freedom.

Case-3:-



Consider an arrangment of five links.

$l = 5$

$p = 5$

$j = 5$

from eq^n-(1)

$l = 2p - 4$

$5 = 2 \times 5 - 4 = 6$

L.H.S < R.H.S.

from eq^n - (2)

$j = \frac{3}{2}l - 2$

$5 = \frac{3}{2} \times 5 - 2 = 5.5$

L.H.S. < R.H.S.

Since the arrangment of five links doesn't satisfy the equations & left hand side is less than the right hand side therefore it is not a kinematic chain, such a type of chain is called un constrained chain.

Mechanism :-

→ when one of the links of a kinematic chain is fixed, the chain is known as mechanism.

→ It may be used for tansmitting or transforming motion

→ A mechanism with four links is called simple mechanism /with more than four links is known as Compound mechanism.

→ when a mechanism is required to do some particular type of work, it then becomes a machine

→ The part of mechanism which initially moves with respect to the link or fixed link or frame is known as driver and the part of the mechanism to which the motion is transmitted is called follower.

Inversion of mechanism :-

→ When one link of a kinematic chain is fixed then it is known as mechanism.

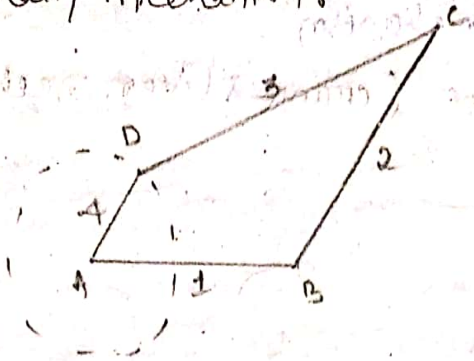
→ We can obtain different mechanism by fixing different links in a kinematic chain.

→ This method of obtaining different mechanism by fixing different links in a kinematic chain is known as inversion of mechanism.

Note:-

The part of a mechanism which initially moves w.r.t the frame or fixed link is known as driver and that part ~~to which~~ of the mechanism to which the motion is transmitted is known as follower.

Four barchain mechanism :-



Grashof's law

→ The simplest & the basic kinematic chain is a four bar chain or quadricycle chain.

→ It consists of four links, each of them forming a turning pair at A, B, C & D.

→ According to Grashof's law, for a four bar mechanism the sum of the shortest & ~~longest &~~ longest link lengths shouldn't be greater than the sum of the remaining of the two links lengths, if there is to be a continuous relative motion betn the two links.

→ In a four bar chain one of the links particularly the shortest link will complete revolution with the other three links if it satisfies the grasof's law.

→ Such a link is known as Crank or driver.

→ The link BC which makes the partial rotation or oscillates is known as lever or rocker or follower.

→ the Link ~~CB~~ CD which connect the ~~th~~ crank and the lever is known as connecting rod or coupler.

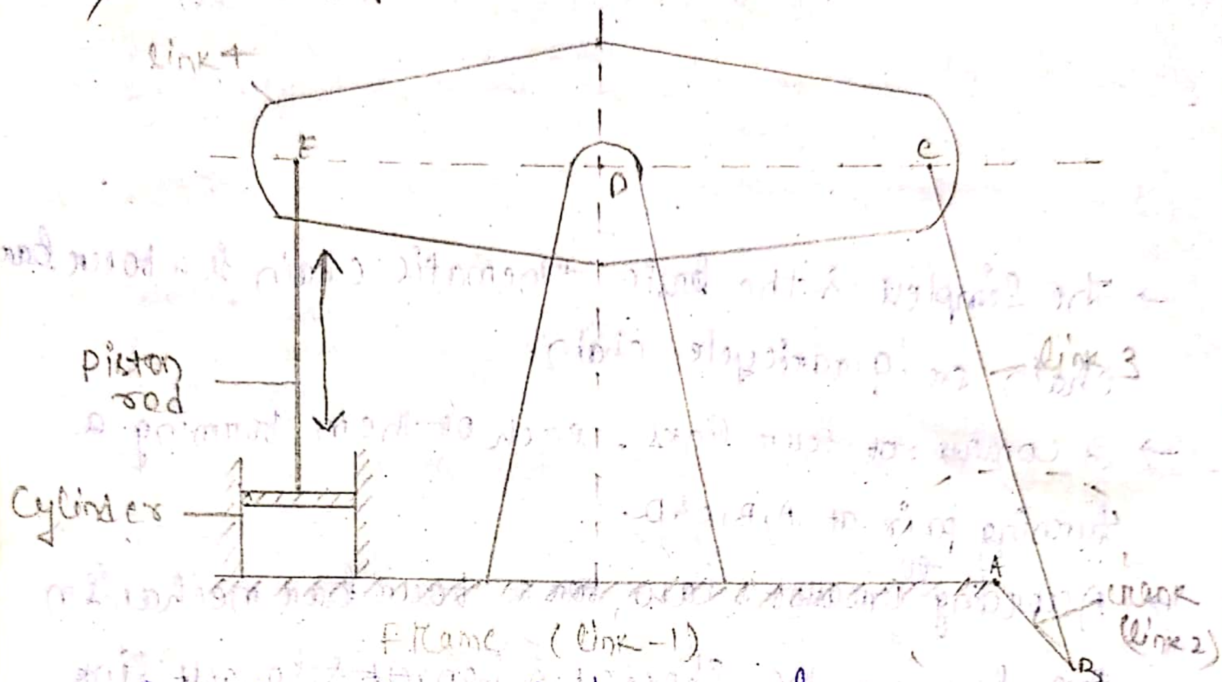
→ The link ~~AB~~ (AB) is known as frame of the mechanism.

→ when the crank is the driver, the mechanism is transforming rotary motion to oscillating motion.

Inversions of a four bar chain mechanism :-

The following are important inversion of a four bar chain mechanism

i) Beam engine (crank & lever mechanism) :-



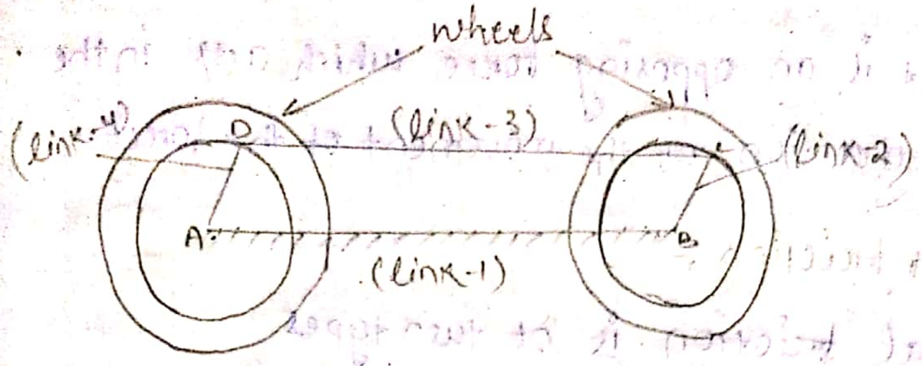
→ A part of the mechanism of a beam engine which consists of four links, is shown in fig. ~~Fig. In this mechanism, when the crank~~

→ In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. ~~The~~

→ The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank.

→ In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

Coupling rod of a locomotive (double crank mechanism) :-



→ The mechanism of a coupling rod of a locomotive which is also known as double crank mechanism consists of four links as shown in the figure.

→ In this mechanism the links AD & BC having equal lengths act as cranks and are connected to the respective wheels.

→ The link CD act as a coupling rod & the length AB is fixed in order to mentain a constant centre to centre distance betn them.

→ This mechanism is ment for transmitting rotary motion from one wheel to another.

Chapter-2 :- Friction :-

Def^n :- It is an opposing force which acts in the opposite direction of the movement of the force.

Types of friction :-

In general friction is of two types.

1. Static friction :-

It is the friction experienced by the body when at rest.

2. Dynamic friction :-

→ It is the friction experienced by the body when in motion.

→ The dynamic friction is also known as Kinetic friction.

→ It is of three types.

1. Sliding friction :-

It is the friction experienced by a body when it slides over another body.

II. Rolling friction :-

It is the friction experienced bet^n the surface which has balls or rollers interposed bet^n them

III. pivot friction :-

It is the friction experienced by a body due to motion of rotation as in case of foot step bearing.

★ Screw friction :-

→ The friction which is experienced in a sqcrew thread, nuts, bolts, studs etc. is known as screw friction.

→ If the threads are cut on the outer surface of a solid rod then the threads are known as external threads but if the threads are cut on the internal surface of a hollow rod then they are known as internal threads.

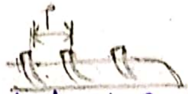
Terminology used in screw friction :-

1. Helix :-

→ It is the curve traced by a particle while describing a circular path which advanced axially at a uniform rate.

→ In other words it is the curve traced by a particle while moving along a screw thread.

2. pitch :-



It is the distance from a point of a screw to a corresponding point on the next thread measured parallel to the axis of the screw.

3. Lead :-

It is the distance a screw thread advance axially in one turn.

4. Depth of thread :-

→ It is the distance betn the top & the buttom surface of the thread.

→ The top surface is known as crest & the buttom surface is known as root.

5. Single threaded screw :-

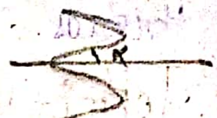
If the lead of a screw is equal to it's pitch, it is known as single threaded screw.

6. Multithreaded screw :-

If more than one thread is cut in one lead distance of a screw then it is known as multithreaded screw.

Mathematically,

lead = pitch × no. of threads.



7. Helix angle :-

It is the slop or inclination of the thread with the horizontal.

Mathematically, $\tan\alpha = \dfrac{\text{lead of screw}}{\text{Circumference of the screw}}$

$= \dfrac{p}{\pi d}$ (for single thread screw)

$= \dfrac{np}{\pi d}$ (for multi threaded screw)

Where, α = Helix angle

p = pitch of the screw

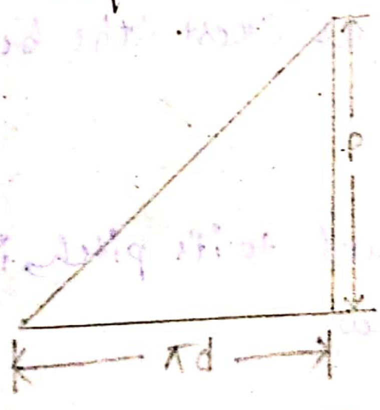
d = mean diameter of the screw

n = no. of threads in one lead.

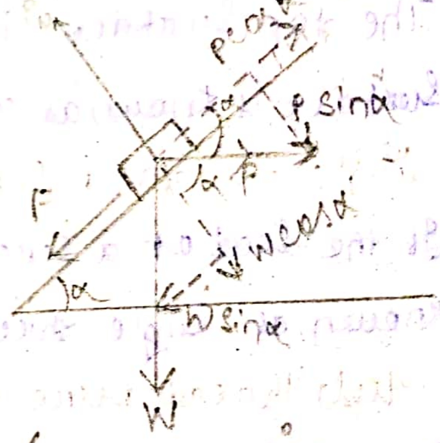
Screw jack :-

Screw jack is a device for lifting heavy loads by applying a comparatively smaller effort at the handle

Torque required to lift the load by a screw jack :



(Development of a screw)



(Force acting on a screw)

Let p = pitch of the screw

d = Mean diameter of the screw

α = Helix angle of the screw

P = effort applied at the circumference of the screw to lift the load.

W = Load to be lifted.

μ = co-efficient of friction betn the screw & nut.

$\mu = \tan\phi$

where ϕ is the friction angle.

From the geometry of the figure, we know that,

$$\tan\alpha = \frac{p}{\pi d}$$

Resolve the forces along the plane.

$$P\cos\alpha = F + W\sin\alpha \quad -(1)$$

Resolve the forces along the $\perp$ to the plane.

$$R_N = W\cos\alpha + P\sin\alpha \quad -(2)$$

from eqn-(1),

$$P\cos\alpha = F + W\sin\alpha$$

$$P\cos\alpha = \mu R_N + W\sin\alpha \qquad (\because F = \mu R_N)$$

~~Pcosα~~ Substitute the value of R_N in eqn (1),

$$P\cos\alpha = \mu W\cos\alpha + \mu P\sin\alpha + W\sin\alpha$$

$$P\cos\alpha - \mu P\sin\alpha = \mu W\cos\alpha + W\sin\alpha$$

$$P(\cos\alpha - \mu\sin\alpha) = W(\mu\cos\alpha + \sin\alpha)$$

$$P = W\,\frac{\mu\cos\alpha + \sin\alpha}{\cos\alpha - \mu\sin\alpha}$$

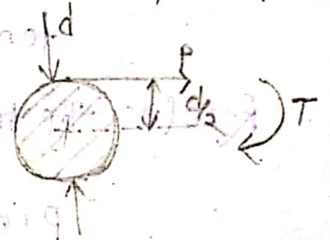
$$= W \times \frac{\sin\alpha + \tan\phi \cos\alpha}{\cos\alpha - \tan\phi \cdot \sin\alpha}$$

$$= W \times \frac{\sin\alpha \cdot \cos\phi + \sin\phi \cos\alpha}{\cos\alpha \cdot \cos\phi - \sin\phi \cdot \sin\alpha}$$

$$= W \times \frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)}$$

$$P = W \times \tan(\alpha+\phi)$$

Torque required to overcome friction between Screw & nut.



$$T_1 = P \times \frac{d}{2}$$

$$= W \tan(\alpha+\phi) \times \frac{d}{2}$$

$$= \frac{dW}{2} \cdot \tan(\alpha+\phi)$$

Torque required to overcome friction at the collar.

$$T_2 = \mu_1 \times W \left(\frac{R_1 + R_2}{2}\right)$$

$$= \mu_1 \times W \times R$$

where R_1 & R_2 are the outside & inside radii of the collar.

R = Mean radius of the collar

μ_1 = Co-efficient of friction of the collar

therefore torque required to overcome the friction

$$T = T_1 + T_2$$

$\Rightarrow T = P \times \frac{d}{2} + \mu_1 W \times R$

If an effort P_1 is applied at the end of a leaver of arm length 'l', then the total required to overcome friction must be equel to the total torque applied at the end of the leaver.

$$\therefore T = P \times \frac{d}{2} = P_1 \times l$$

Note:-

when the nominal die ~~do~~ (d_o) and the core die (d_c) of this screw thread is given then the mean die of the screw, $d = \frac{(d_o + d_c)}{2} = d_o - \frac{p}{2} = d_c + \frac{p}{2}$

Since the mechanical advantage is the ratio of ~~load~~ lifted (W) to the effort applied (P_1) at the end of the leaver therefore,

$$MA = \frac{W}{P_1} \qquad \left(\because P_1 = \frac{Pd}{2l}\right)$$

$$= \frac{W \times 2l}{Pd}$$

$$= \frac{\cancel{W} \times 2l}{\cancel{W} \tan(\alpha+\phi) d}$$

$$\boxed{MA = \frac{2l}{d \tan(\alpha+\phi)}}$$

Problem :-

1. An electric motor driven screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm/min. The screw has a single square thread of 6 mm pitch on a major dia of 40 mm. The coefficient of friction at the screw thread is 0.1. Estimate power of the motor.

Given data,

$W = 75\ kN$

$d_o = 40\ mm$

$p = 6\ mm$

$\mu = 0.1 = \tan\phi$

$N = 300\ mm/min.$

$N = \frac{300}{6}\ rpm = 50\ rpm$

We know,

$$\text{Power of the motor} = \frac{2\pi N T}{60}$$

$\cancel{T = W\tan(\alpha+\phi)}$ $\quad T = P \times \frac{d}{2}$

$\cancel{T = P \times \frac{d}{2}}$

$$d = d_o - \frac{p}{2} = 40 - \frac{6}{2} = 37\ mm = 37\times10^{-3}\ m$$

$$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 37} = 0.0516$$

$$P = W\tan(\alpha+\phi)$$

~~Pow~~

$$= 75\times10^{3} \times \frac{(\tan\alpha + \tan\phi)}{1 - \tan\alpha\cdot\tan\phi}$$

$$= 75\times10^{3} \times \frac{0.0516 + 0.1}{1 - 0.0516\times0.1}$$

$$= 11.42\times10^{3}\ N$$

$$T = P\times\frac{d}{2} = 11.42\times10^{3}\times\frac{37\times10^{-3}}{2} = 211.43\ kN\text{-}m$$

Scanned with CamScanner

Power of the motor $= \frac{2\pi NT}{60}$

$= \frac{2\pi \times 50 \times 211.43}{60}$

$= 1.107 \times 10^3$ W

$= 1.107$ KW

$N = \frac{\text{Speed of the nut}}{\text{Pitch of the screw}}$

Power = 1.108 KW

2. A square thread bolt of root diameter 22.5 mm & pitch 5 mm is tightened by screwing a nut whose mean diameter of bearing surface is 50 mm. If co-efficient of friction for nut & bolt is 0.1 & for nut bearing surface is 0.16, find the force required at the end of a spanner 500 mm long when the load on the bolt is 10KN.

Given data, $d_c = 22.5$ mm, $l = 500$ mm

$p = 5$ mm, $W = 10$ KN

$D = 50$ mm

$\mu = 0.1$, $d = d_c + p/2 = 22.5 + 2.5 = 25$ mm

$\mu_1 = 0.16$

$\tan\alpha = \frac{p}{\pi d} = \frac{5}{\pi \times 25} = \cancel{0.0318}\ 0.0636$

$P = W \tan(\alpha + \phi)$

$= 10 \times 10^3 \times \frac{0.0636 + 0.1}{1 - 0.0636 \times 0.1}$

$= 1646.47$ N

$T = P \times \frac{d}{2} + \mu_1 \times W \times R$

$= 1646.47 \times \frac{25}{2} + 0.16 \times 10 \times 10^3 \times \frac{50}{2}$

$= 60580.875$ N-m

$T = P_1 \times l$

$\Rightarrow P_1 = \frac{T}{l}$

$= \frac{60580.875}{0.5}$

$= 121161.7N\ N$

$\Rightarrow P_1 = 121.161 KN$

3. A 150 mm dia valve against which a steam pressure of $2 MN/m^2$ ~~for a square~~ is acting, is closed by means of a square thread screw. 50 mm external dia with 6 mm pitch. If the coefficient of friction is 0.12 find the ~~torque~~ torque required to turn the handle.

$MN = Mega Netr$ Press = $2 MN = 2 \times 10^6 N$

$d_0 = 50 mm$

$p = 6 mm$

$\mu = 0.12 = \tan\phi$

$W = Pr. \times Area$

$= 2 \times 10^6 \times \frac{\pi}{4} \times .15^2$

$= 35342.91 \simeq 35342$

$d = d_0 - 2\frac{p}{2}$

$= 50 - \frac{6}{2}$

$= 47 mm$

$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 47} = 0.0406$

$\tan\phi = \mu = 0.12$

$P = W\tan(\alpha + \phi)$

$= 35343 \times 0.161 = 5703.87 N$

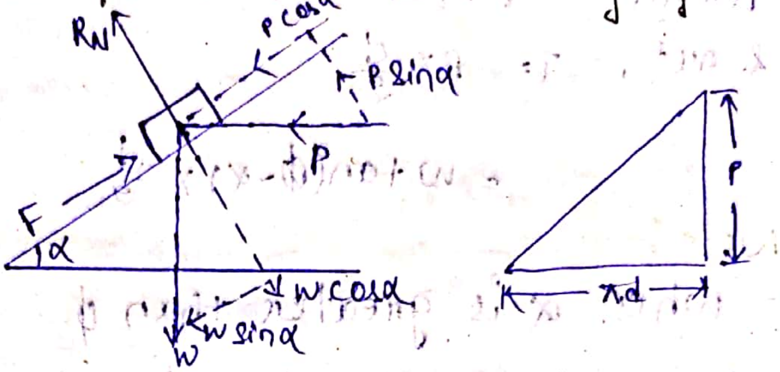
Torque required to turn the handle 'T' is given by

$$T = p \times \frac{d}{2}$$

$$= 5763.87 \times \frac{47 \times 10^{-3}}{2}$$

$$= 134\ Nm$$

Torque required to lower the body by screw jack:



From the geometry of the figure we find that

$$\tan\alpha = \frac{p}{\pi d}$$

Resolve the forces along the plane

$$P\cos\alpha = F - W\sin\alpha$$

$$P\cos\alpha = \mu R_N - W\sin\alpha \quad -(1)$$

Resolve the forces perp $\perp^r$ to the plane

$$R_N = W\cos\alpha - P\sin\alpha \quad -(11)$$

Substitute the R_N value in eq^n (1), we get,

$$P\cos\alpha = \mu W\cos\alpha - \mu P\sin\alpha - W\sin\alpha$$

$$P\cos\alpha + \mu P\sin\alpha = \mu W\cos\alpha - W\sin\alpha$$

$$P(\cos\alpha + \mu\sin\alpha) = W(\mu\cos\alpha - \sin\alpha)$$

$$P = \frac{W(\mu\cos\alpha - \sin\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$

Substitute the value $\mu = \tan\phi$ in the above eq^n.

$$P = \frac{W(\tan\phi\cos\alpha - \sin\alpha)}{(\cos\alpha + \tan\phi\sin\alpha)}$$

$$P = \frac{W(\sin\phi\cos\alpha - \cos\phi\sin\alpha)}{\cos\alpha\cdot\cos\phi + \sin\phi.\sin\alpha}$$

$$P = \frac{W\sin(\phi-\alpha)}{\cos(\phi-\alpha)}$$

$$P = W.\tan(\phi-\alpha)$$

Torque required to overcome friction betn screw & nut, $T = P \times \frac{d}{2}$

$$= W\tan(\phi-\alpha) \times \frac{d}{2}$$

Note :-

When α is greater than ϕ

then, $P = W\tan(\alpha-\phi)$

Efficiency of a screw jack :-

The efficiency of a screw jack may be defined as the ratio betn ideal effort (i.e. the effort required to move the load negleting friction) to the actual effort (i.e. the effort required to move the load considering friction).

We know that,

Effort required to move the load when friction is considered, $P = W\tan(\alpha+\phi)$

When friction is not considered then the value of ϕ becomes zero,

therefore, the ~~actual~~ Ideal effort, $P_0 = W\tan\alpha$

Therefore, efficiency $(\eta) = \frac{\text{Ideal}}{\text{actual}} = \frac{W\tan\alpha}{W\tan(\alpha+\phi)}$

$$= \frac{P_0}{P}$$

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Max. efficiency of screw jack :-

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\sin\alpha}{\cos\alpha} \cdot \frac{\cos(\alpha+\phi)}{\sin(\alpha+\phi)}$$

$$= \frac{2\sin\alpha\cos(\alpha+\phi)}{2\cos\alpha\sin(\alpha+\phi)}$$

$$= \frac{\sin(2\alpha+\phi) + \sin(-\phi)}{\sin(2\alpha+\phi) - \sin(-\phi)}$$

~~The maximum should be maximum~~

The efficiency should be max when $(2\alpha+\phi)$ is maximum.

$\therefore$ $\sin(2\alpha+\phi) = 1$

$2\alpha+\phi = 90'$

$2\alpha = 90' - \phi$

$\alpha = \frac{90'-\phi}{2} = 45° - \frac{\phi}{2}$

Substitute the value of 2α in above eq^n..

$$= \frac{\sin(90' - \cancel{\phi} + \cancel{\phi}) - \sin(\phi)}{\sin(90' - \cancel{\phi} + \cancel{\phi}) + \sin\phi}$$

$$\eta_{max} = \frac{\sin 90' - \sin\phi}{\sin 90' + \sin\phi} = \frac{1-\sin\phi}{1+\sin\phi}$$

The mean dia of a square threaded screw jack is 50 mm. The pitch of the thread is 10 mm. The co-efficient of friction is 0.15. What force must be applied at the end of a 0.7 m long lever which is ⊥^{r} to the longitudinal axis of the screw to raise a load of 20 kN & to lower it.

Given data, $d = 50$ mm

$p = 10$ mm

$\mu = 0.15$

$l = 0.7$

$W = 20$ kN

α

$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0636$

For raise a load,

$P = W \tan(\alpha + \phi)$

$= 20 \times 10^3 \times \frac{0.0636 + 0.15}{1 - 0.0636 \times 0.15}$

$= \cancel{0.215 N}$

$= 4313.14 N$

$T = P \times \frac{d}{2}$

$= 4313.14 \times \frac{50 \times 10^{-3}}{2}$

$= 107.82 N\text{-}m$

$T = P_1 \times \frac{d}{2} = P_1 \times l$

$107.82 = P_1 \times 0.7$

$\Rightarrow P_1 = \frac{107.82}{0.7} = 154 N$

For lower a load,

$P = W \tan(\alpha - \phi)$

$= 20 \times 10^3 \times \frac{0.0636 - 0.15}{1 + 0.0636 \times 0.15}$

$= 1711.6 N$

$T = P \times \frac{d}{2} = P_1 \times l$

$1711.6 \times \frac{50 \times 10^{-3}}{2} = P_1 \times 0.7$

$\Rightarrow P_1 = \frac{1711.6 \times 50 \times 10^{-3}}{2 \times 0.7} = 61.12 N$

1. A pitch of 50 mm mean dia of threaded screw Jack is 12.5 mm. The μ betn screw & nut is 0.13. Determine the torque required to raise a load of 25 KN. assuming the load is rotate with screw. determine the ratio of the torque required to raise the load, to the torque required to lower the load & also efficiency of the machine.

Given data, $d = 50$ mm $= 0.05$ m

$p = 12.5$ mm

$\mu = 0.13 = \tan\phi$

$W = 25$ KN

$$\tan\alpha = \frac{p}{\pi d} = \frac{12.5}{\pi \times 50} = 0.0795$$

$$P_{raise} = W \tan(\alpha + \phi)$$

$$= 25 \times 10^3 \times \frac{\tan(0.07 + 0.13)}{1 - 0.0795 \times 0.13}$$

$= \cancel{50485.9\ KN}$ $= 5300$ N

$= \cancel{5.04 \times 10^3}$ N $= 5.3$ KN

$$T_{raise} = P \times \frac{d}{2} = 5.3 \times 10^3 \times \frac{0.05}{2} = \cancel{126.14\ N\text{-}m}\ 132.5\ N\text{-}m$$

$$P_{lower} = W \tan(\alpha - \phi)$$

$$= 25 \times 10^3 \times \frac{\cancel{0.07 - 0.}\ 0.13 - 0.07}{1 - 0.07 \times 0.13}$$

$= \cancel{1513.77\ N}$

$= 1275.68$ N

$$T_{lower} = 1278.68 \times \frac{0.05}{2} = 31.8\ N\text{-}m$$

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$\therefore \frac{T_1}{T_2} = \frac{132.5}{31.8} = 4.15$

efficiency $(\eta) = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{0.0795}{0.211}$

$= 37.6\%$

2. A load of 10 KN is raised by means of a screw jack having a square threaded screw of 12 mm pitch & of mean dia 350mm. If a force of 100 N is applied at the end of the lever to raise the load, what should be the length of the lever is used? Take $\mu = 0.15$. what is M.A. obtained? State weather the screw is self locking.

Given data, $W = 10\,KN$

$P = 12\,mm$

$d = 350\,mm$

$F = 100N$

$\mu = 0.15$

$\tan\alpha = \frac{P}{\pi d} = \cancel{0.25076}\ 0.076$

$\boxed{\alpha = \tan^{-1}(0.25) = 14.28}$

B. $\tan\phi = 0.15$

$\phi = 8.53$

$\boxed{P = 10\times10^3 \times \tan(14.28 + 8.53)}$

$P = 10\times10^3 \times \frac{0.076 + 0.15}{1 - 0.076\times0.15}$

$= 2286\,N$

$$T = 2286 \times \frac{50 \times 10^{-3}}{2}$$

$$= \cancel{13.41\ N\text{-}m}$$
$$= 57.15\ N\text{-}m$$

$$T = P \times \frac{d}{2} = P_1 \times l$$

$$57.15 = 100 \times l$$

$$l = 0.5715\ m$$

$$= 571.5\ mm$$

$$MA = \frac{W}{P} = \frac{10 \times 10^3}{100} = 100$$

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{0.076}{0.2286}$$

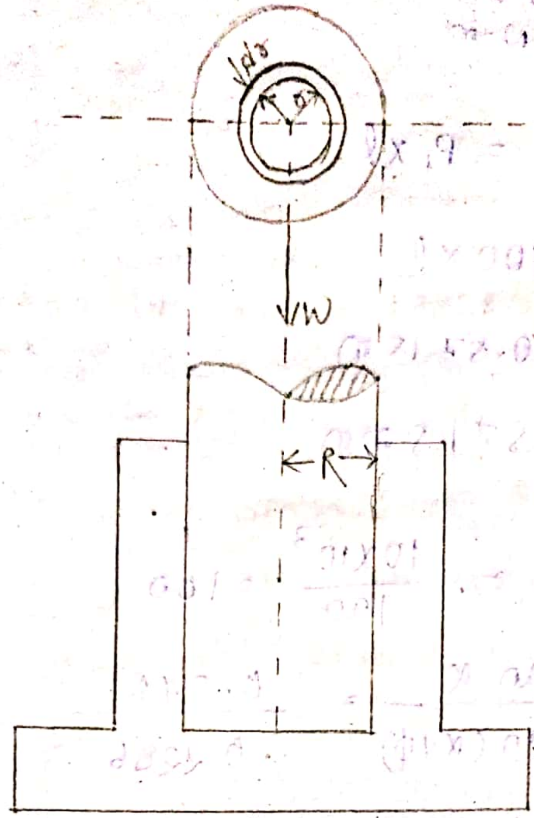
$$= 33.24\ \%$$

Hence the efficiency is less than 50% (i.e. 33.24%) therefore it is self locking screw.

$\eta > 50\%$ (Overhaul machine)

$\eta < 50\%$ (Self locking machine)

Flat pivot bearing :-



← Flat pivot bearing

When a vertical shaft rotates in a flat pivot bearing known as footstep bearing, the sliding friction will be along the surface of contact betn the shaft & bearing surface.

Let, W = Load transmitted over the bearing surface.

R = Radius of the bearing surface

p = Intensity of pressure per unit area of the bearing surface.

μ = Co-efficient of friction.

Consider the following two cases.

1. When there is a uniform pressure

II. When there is a uniform wear.

Case -1

Considering uniform pressure :-

When the pressure is uniformly distributed over the bearing area then

$$p = \frac{W}{\pi R^2}.$$

Consider a ring of radius 'r' & thickness 'dr' of the bearing area.

$\therefore$ Area of the bearing surface.

$$A = 2\pi r.dr$$

load transmitted to the ring

$$\delta W = p \times A$$
$$= p \times 2\pi r.dr$$

Frictional resistance to sliding on the ring acting tangentially at radius 'r'.

$$F_r = \mu \times \delta W \qquad \mu = \frac{F}{R}$$
$$= \mu \times p \times 2\pi r.dr$$

Frictional torque on the ring $(T_r) = F_r \times r$

$$= \mu p\, 2\pi r.dr \times r$$
$$= \mu p.\, 2\pi r^2.dr$$

Integrating the eqn within the limits from 0 to R for the total torque on the pivot bearing.

$$T = \int_0^R \mu p\, 2\pi r^2 dr$$

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$$T = \mu p . 2\pi . \int_0^R r^2 . dr$$

$$= \mu p . 2\pi . \left[\frac{r^3}{3}\right]_0^R$$

$$= \mu p . 2\pi \left[\frac{R^3}{3} - \frac{0^3}{3}\right]$$

$$= \mu p . 2\pi \frac{R^3}{3}$$

$$T = \frac{2}{3} . \mu p \pi R^3$$

Put the value of $p = \frac{W}{\pi R^2}$ in above equation.

$$\therefore T = \frac{2}{3} . \mu . \frac{W}{\cancel{\pi} \cancel{R^2}} \times \cancel{\pi} \times R^{\cancel{3}}$$

$$\boxed{T = \frac{2}{3} . \mu W R}$$

Case-2

considering a uniform wear :

- It is assumed that the rate of wear is proportional to the product of intensity of pressure & the velocity of the rubbing surface.
- Since the velocity of the rubbing surface increases with the distance from the axis of the bearing, therefore for the uniform wear

$$p \times r = C$$

$$p = \frac{C}{r}$$

Load transmitted on the ring, $\delta W = p \times 2\pi r . dr$

$$= \frac{c}{r} \times 2\pi r . dr$$

$$= 2\pi c\, dr$$

The total load transmitted to the bearing surface.

$$W = \int_0^R \delta W = \int_0^R 2\pi c\, dr$$

$$= 2\pi c \int_0^R dr$$

$$= 2\pi c [r]_0^R$$

$$= 2\pi c (R-0)$$

$$W = 2\pi c R$$

$$\therefore c = \frac{W}{2\pi R}$$

$c = p \times r$

We know that the frictional torque (F_R) acting on the ring

$$T_r = 2\pi \mu p r^2 dr$$

$T = (2\pi c\, dr) \times r$

$$= 2\pi \mu . \frac{c}{r} r^2 dr$$

$$= 2\pi \mu c r\, dr$$

Now the total frictional torque bearing on the surface.

$$T = \int_0^R 2\pi \mu c r\, dr$$

$$= 2\pi \mu c \left[\frac{r^2}{2}\right]_0^R$$

$$= \cancel{2}\pi \mu c \frac{R^2}{\cancel{2}}$$

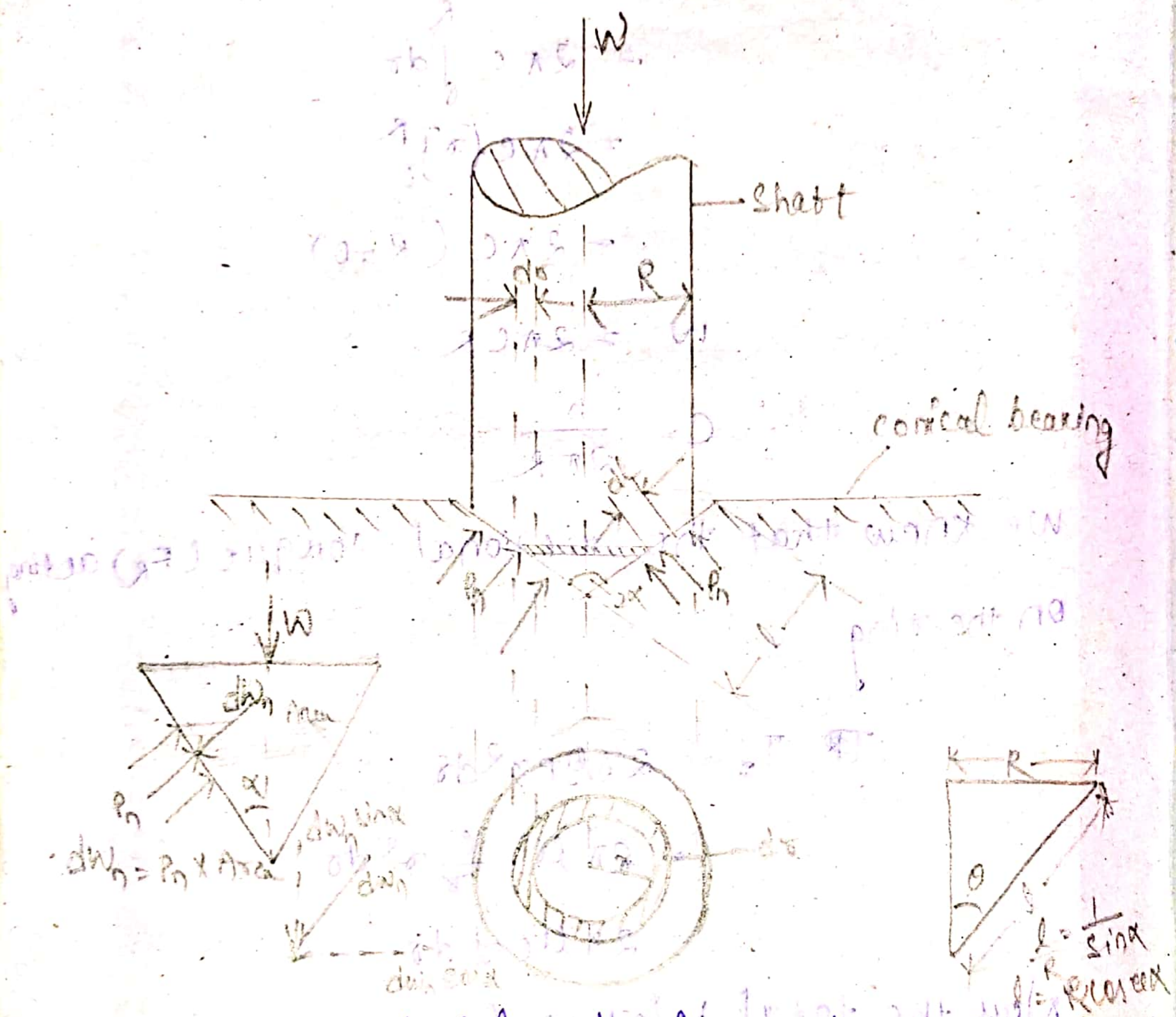
$\Rightarrow T = \pi \mu c R^2$

Put the value of c in above eqn.

$\Rightarrow T = \pi \mu . \frac{W}{2\pi R} \times R^2$

$\boxed{T = \frac{1}{2} \mu . W . R}$

Conical pivot Bearing :-



The conical pivot bearing supporting a shaft carrying a load 'W' as shown in the figure.

let, P_n = Intensity of the pressure normal to the cone.

α = Semi angle of the cone.

μ = Co-efficient of friction betn the shaft & the bearing.

R = Radius of the shaft

Consider a small ring of radius 'r' and thickness 'dr'.

let 'dl' is the length of the ring along the cone such that, $dl = dr \operatorname{cosec} \alpha$.

$\therefore$ Area of the ring, $A = 2\pi r \cdot dl$

$= 2\pi r \cdot dr \operatorname{cosec} \alpha$

$(\because dl = dr \operatorname{cosec} \alpha)$

Case-1

considering uniform pressure :-

We know that normal load acting on the ring, is equal to δW_n = normal pressure × area.

i.e. $\delta W_n = P_n \times \text{Area}$

$= P_n \times 2\pi r \cdot dr \operatorname{cosec} \alpha$

Case-2

~~vertical load on the ring :-~~

Vertical load acting on the ring is

δW = vertical component of δW_n.

$\therefore \delta W = \delta W_n \sin\alpha$

$= P_n \times 2\pi r \cdot dr \operatorname{cosec}\alpha \cdot \sin\alpha$

$= P_n \times 2\pi r \, dr$

Total vertical load transmitted to the bearing.

$$W = \int_0^R P_n \, 2\pi r \, dr$$

$$= P_n \cdot 2\pi \left[\frac{r^2}{2}\right]_0^R = P_n 2\pi \frac{R^2}{2}$$

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$W = P_n \pi R^2$

$\Rightarrow P_n = \dfrac{W}{\pi R^2}$

Frictional force on the ring acting tangentially at radius 'r'.

$F_r = \mu \times \delta W_n$

$= \mu \times P_n \times 2\pi r \, dr \operatorname{cosec}\alpha$

Frictional torque acting on the ring,

$T_r = F_r \times r$

$= \mu \times P_n 2\pi r \, dr \operatorname{cosec}\alpha \times r$

$= \mu P_n 2\pi r^2 dr \operatorname{cosec}\alpha$

$= 2\pi \mu P_n \operatorname{cosec}\alpha \, r^2 dr$

Now the total torque can be found out by integrating the above expression from limit '0' to 'R'.

$T = \int_0^R T_r = \int_0^R 2\pi \mu P_n \operatorname{cosec}\alpha \, r^2 dr$

$= 2\pi \mu P_n \operatorname{cosec}\alpha \left[\dfrac{r^3}{3}\right]_0^R$

$= 2\pi \mu P_n \operatorname{cosec}\alpha \, \dfrac{R^3}{3}$

$T = \dfrac{2}{3}\pi \mu P_n \operatorname{cosec}\alpha \, R^3 \quad -(1)$

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Substitute the value of P_n in eq^n-(i) we get,

$$T = \frac{2}{3}\cancel{\pi}\mu \times \frac{W}{\cancel{\pi}\cancel{R^2}} \times \operatorname{cosec}\alpha\, R^{\cancel{3}}$$

$$= \frac{2}{3}\mu W \operatorname{cosec}\alpha R$$

$$= \frac{2}{3}\mu W R\operatorname{cosec}\alpha.$$

$$\boxed{T = \frac{2}{3}\mu W l}$$ ($\because R\operatorname{cosec}\alpha = l$)

Case -2 :-

Considering uniform wear :-

Let 'P_r' be the normal intensity of pressure at a distance 'r' from the central axis.

$$P_r \times r = C$$

$$\Rightarrow P_r = \frac{C}{r}$$

load transmitted to the ring, $\delta w = \cancel{\delta w_n \sin} P_r \times 2\pi r dr$

$$\delta w = \frac{C}{\cancel{r}} \cdot 2\pi \cancel{r} dr$$

$$\delta w = 2\pi C dr$$

Total load transmitted to the ring,

$$W = \int_0^R \delta w = \int_0^R 2\pi C dr.$$

$$= 2\pi C[r]_0^R$$

$$W = 2\pi C R$$

$$\Rightarrow C = \frac{W}{2\pi R}$$

Frictional torque acting on the ring,

$$T_r = \mu p_r 2\pi r^2 dr \operatorname{cosec}\alpha$$

Put the value of p_r in above eqn we get.

$$T_r = \mu \times \frac{c}{r} \cdot 2\pi r^2 dr \operatorname{cosec}\alpha$$

$$= \mu c\, 2\pi dr \operatorname{cosec}\alpha \cdot r$$

$$= 2\pi \mu c \operatorname{cosec}\alpha \cdot r\, dr$$

Total torque acting on the ring,

$$T = \int_0^R T_r = \int_0^R 2\pi \mu c \operatorname{cosec}\alpha\, r\, dr$$

$$= 2\pi \mu c \operatorname{cosec}\alpha \left[\frac{r^2}{2}\right]_0^R$$

$$T = \pi \mu c \operatorname{cosec}\alpha\, R^2 \quad -(1)$$

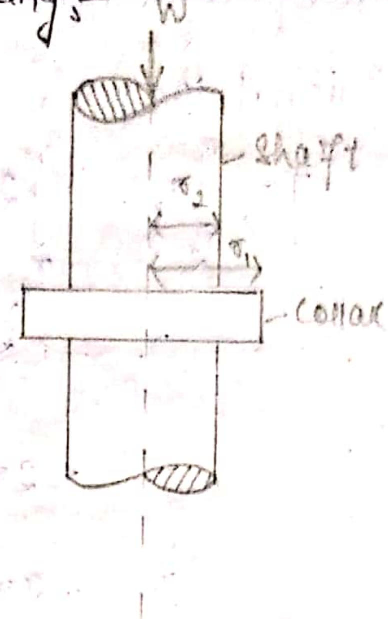
Put the value of c in the above eqn, we get

$$T = \pi \cdot \mu \cdot \frac{W}{2\pi R} \operatorname{cosec}\alpha\, R^2$$

$$= \boxed{\mu \frac{W}{2} R \operatorname{cosec}\alpha}$$

$$= \boxed{\frac{1}{2}\mu W} = \frac{1}{2}\mu W l$$

Flat collar bearing :-



Consider a single flat collar bearing supporting a shaft as shown in the fig.

Let,
r_1 = External radius of the collar
r_2 = Internal radius of the collar.

Area of the bearing surface $(A) = \pi [r_1^2 - r_2^2]$

Case-1

Considering uniform pressure :-

Intensity of pressure, $p = \frac{W}{A}$

$$p = \frac{W}{\pi [r_1^2 - r_2^2]}$$

As we know, the frictional torque on the ring of radius 'r' + thickness 'dr',

$$T_r = 2\pi \mu p \cdot r^2 \, dr$$

Integrating the eq^n with in the limit r_2 to r_1, then the total frictional torque,

$$T = \int_{r_2}^{r_1} T_r = \int_{r_2}^{r_1} 2\pi\mu p r^2\, dr$$

$$= \cancel{\frac{2}{3}\pi\mu p R^3}$$

$$= \cancel{2\pi\mu p R^3}$$

$$T = 2\pi\mu p \left[\frac{r_1^3 - r_2^3}{3}\right] \quad -(1)$$

Substitute the value of p in eq^n-(1), we get

$$\Rightarrow T = 2\pi\mu \frac{W}{\pi(r_1^2 - r_2^2)} \times \frac{(r_1^3 - r_2^3)}{3}$$

$$\Rightarrow T = \frac{2}{3}\mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

Note :-

i) In case of a multi collar bearing say 'n' no. of collars, then intensity of uniform pressure,

$$P = \frac{W}{A}$$

$$\Rightarrow P = \frac{W}{n\pi[r_1^2 - r_2^2]}$$

ii) The total torque transmitted in multicollar bearing remains Constant.

i.e. $$T = \frac{2}{3}\mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

Case-2

Considering uniform wear :-

Load transmitted on the ring considering uniform wear.

$$\delta w = P_r \times 2\pi r \times dr$$

$$= \frac{c}{r} \times 2\pi r \times dr$$

$$= 2\pi c\, dr$$

Total load transmitted to the collar by integrating from r_2 to r_1.

$$W = \int_{r_2}^{r_1} 2\pi c\, dr$$

$$W = 2\pi c (r_1 - r_2)$$

$$\Rightarrow c = \frac{W}{2\pi (r_1 - r_2)}$$

Frictional torque on the ring,

$$T_r = F_r \times r$$

$$= \mu \times \delta w \times r$$

$$= \cancel{2\pi \mu c\, dr \times r\, dr}$$

$$= 2\pi \mu c\, r \cdot dr$$

Total torque on the bearing.

$$T = \int_{r_2}^{r_1} T_r = \int_{r_2}^{r_1} 2\pi \mu c\, r\, dr$$

$$= 2\pi \mu c \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi \mu c \frac{(r_1^2 - r_2^2)}{2}$$

$$= \pi \mu c (r_1^2 - r_2^2)$$

Substitute the value of c in the above eq?.

$T = \mu \pi c \, (r_1^2 - r_2^2)$

$= \mu \cancel{\pi} \times \frac{W}{2\cancel{\pi}(r_1 - \cancel{r_2})} \times (r_1^2 - \cancel{r_2^2})$

$= \frac{1}{2} \mu W \, (r_1 + r_2)$

Question :-

A thrust shaft of a ship has 6 collars of 600mm external dia & 300mm internal dia. The total thrust of propeller is 100 KW If co-efficient of friction is 0.12, if speed of the engine is 90 rpm. Find the power absorbed in the friction. At the thrust block assuming

i) uniform pressure

ii) uniform wear.

Given data, $d_1 = 600mm = 0.6 m$ $W = 100KW$

$d_2 = 300mm = 0.3 m$ $\mu = 0.12$

$N = 90$ rpm

$\Rightarrow r_1 = 0.3m$

$\Rightarrow r_2 = 0.15m$

Power (P) $= \frac{2\pi N T}{60}$

For uniform pressure;

$$T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$= \frac{2}{3} \times 0.12 \times 100 \times 10^3 \times \left[\frac{0.3^3 - 0.15^3}{0.3^2 - 0.15^2}\right]$$

$= 2800$ N-m

power $= \frac{2\pi N T}{60} = \frac{2\pi \times 90 \times 2800}{60} = 26.38$ KW.

For uniform wear;

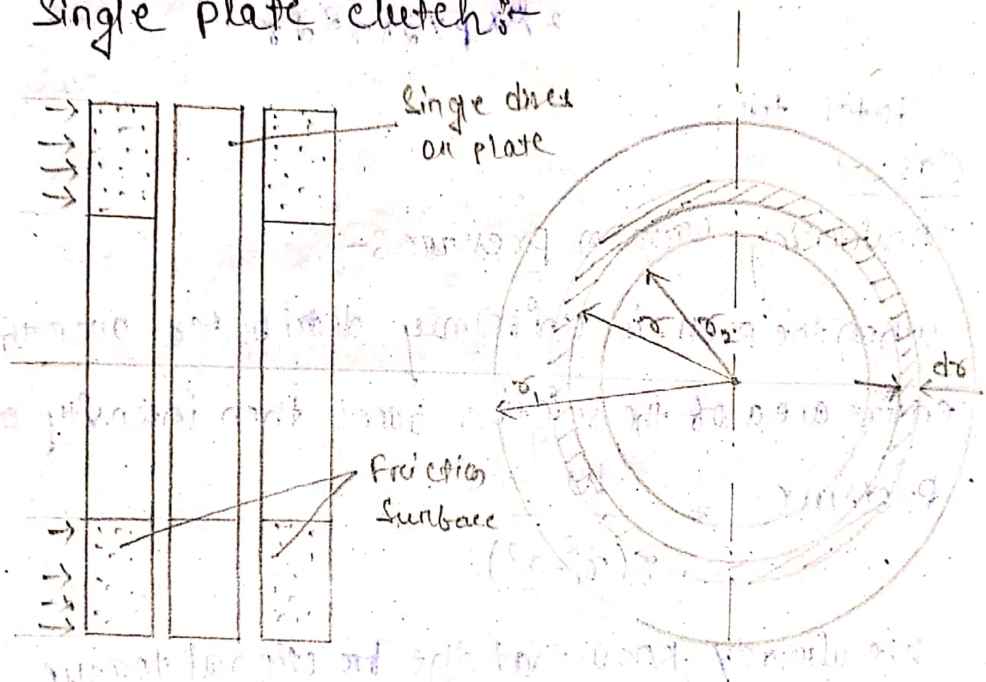
$$T = \frac{1}{2} \mu W (r_1 + r_2)$$

$$= \frac{1}{2} \times 0.12 \times 100 \times 10^3 \times (0.3 + 0.15)$$

$$= 2700 \text{ N-m}$$

$$P = \frac{2\pi \times 90 \times 2700}{60} = 25.44 \text{ kW}$$

Single plate clutch :-



Let, T = Torque transmitted by the clutch

p = Intensity of axial press. with which the contact surface are held together.

r_1 & r_2 = Exter. & inter. radii of the friction surface

μ = co-efficient of friction

Consider an elementary ring of radius r & thickness 'dr'.

We know that Area of contact surface or friction surface = $2\pi r\, dr$

Normal force on the ring, δw = $p \times A$

$= p \times 2\pi r dr$

Frictional force on the ring acting tangentially at radius r

$F_r = \mu \times \delta W$

$= \mu \times p \times 2\pi r\, dr$

Fritional torque acting on the ring,

$T_r = F_r \times r$

$= \mu \times p \times 2\pi r^2 dr$

~~Total torq~~

Case-1

considering uniform pressure :-

when the press. is uniformly distributed over the entire area of the friction faces then intensity of

Pressure $= \dfrac{W}{\pi(r_1^2 - r_2^2)}$

We already know that the frictional torque,

$T_r = \mu p 2\pi r^2 dr$

Intigrating the above eqn from limit r_2 to r,

$$\therefore T = \int_{r_2}^{r_1} T_r = \int_{r_2}^{r_1} 2\pi \mu p\, r^2 dr$$

$$= 2\pi\mu p\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= \frac{2}{3}\pi\mu p\left(r_1^3 - r_2^3\right) \quad \text{---(1)}$$

Substitute the value of P in eqn -(1), we get,

$$\therefore T = \frac{2}{3} \times \pi \cdot \mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \cdot (r_1^3 - r_2^3)$$

$= \frac{2}{3} \mu W \frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)}$

$= \mu W R$

Where, $R =$ Mean radius of friction surface

$= \frac{2}{3} \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)$

Case-2

Considering uniform wear:-

let p be the normal intensity of press at a distance 'r' from the axis of clutch.

We know that, $p \times r = c$

$\Rightarrow p = \frac{c}{r}$

Normal force acting on the ring,

$\delta w = p \times 2\pi r \, dr$

$= \frac{c}{\cancel{r}} \times 2\pi \cancel{r} \, dr$

$= 2\pi c \, dr$

Total frictional force,

$W = \int_{r_2}^{r_1} \delta w = \int_{r_2}^{r_1} 2\pi c \, dr$

$W = 2\pi c (r_1 - r_2)$

$\Rightarrow c = \frac{W}{2\pi (r_1 - r_2)}$

we know, frictional torque acting on the ring,

$T_r = 2\pi \mu p r^2 \, dr$

$= 2\pi \mu \cdot \frac{c}{r} \times r^2 dr$

$= 2\pi \mu c r \, dr$

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} T_r = \int_{r_2}^{r_1} 2\pi \mu c \, r \, dr$$

$$= \cancel{2}\pi \mu c \left[\frac{r^2}{\cancel{2}}\right]_{r_2}^{r_1}$$

$$= \pi \mu c (r_1^2 - r_2^2)$$

Put the value of c in above eqn, we get,

$$\Rightarrow T = \cancel{\pi} \times \mu \times \frac{W}{2\cancel{\pi}(r_1 - \cancel{r_2})} \times (r_1^2 - r_2^2)$$

$$T = \frac{1}{2} \mu W (r_1 + r_2)$$

$$= \mu W R$$

where R = Mean radius of friction surface whose ~~value is~~
value is $\frac{r_1 + r_2}{2}$

Note :-

i) In general the total frictional torque (T_r) acting on the friction surface is given by,

$$T = n \mu W R$$

where, n = no. of pairs of friction surface.

R = Mean radius of the friction surface

$$= \frac{2}{3} \times \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right) \text{ (for uniform pressure)}$$

$$= \left(\frac{r_1 + r_2}{2}\right) \text{ (for uniform wear)}$$

ii) For single plate clutch, both sides of the discs are effective. Therefore single disc clutch has two pairs of contact surface.

i.e. $n = 2$

iii) Since the intensity of pressure is max. at the inner radius 'r_2', therefore the eqn may be written as, $P_{max} \times r_2 = c$

iv) Since the intensity of pressure is min. at the outer radius 'r_1', therefore the eqn may be written as,

$$P_{min} \times r_1 = c$$

Multiple disc clutch :-

Let, n_1 = No. of discs on the driving shaft

n_2 = No. of discs on the driven shaft

∴ No. of pairs of contact surface,

$$n = n_1 + n_2 - 1$$

Total frictional torque, $T = n \mu W R$

Question :-

A single plate clutch with both sides effective has outer & inner diameters 300mm & 200mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed $0.1 N/mm^2$. If the co-efficient of friction is 0.3, determine the power transmitted by clutch at a speed 2500 rpm.

~~n=2~~ Given data, $n=2$

$d_1 = 300mm$, $r_1 = 150mm$

$d_2 = 200mm$, $r_2 = 100mm$

$P = 0.1 N/mm^2$

$\mu = 0.3$ $P = ?$

$N = 2500 rpm$

Since the intensity of pres. is max. at the inner radius, therefore for conform wear, $P \times r_2 = C$

$\Rightarrow C = 0.1 r_2 = 0.1 \times 100 = 10 mm$

$W = 2\pi C (r_1 - r_2) = 2\pi . 10 (150-100) = 3142 N$.

Mean radius,

$$R = \frac{r_1 + r_2}{2} = \frac{150+100}{2} = 125 mm = 0.125 m$$

$$T = n\mu W R = 2 \times 0.3 \times 3142 \times 0.125$$
$$= 235.65 N\text{-}m.$$

Power transmitted by clutch,

$$P = \frac{2\pi NT}{60} = 61.693 kW$$

2. A single plate clutch effective on both sides is required to transmit 25 kW at 3000 rpm. Determine the outer and inner radii of frictional surface, if co-efficient of friction 0.255, the ratio of radii is 1.25 & max. pres. is not to exceed $0.1 N/mm^2$. Also determine the axial thrust to be provided by the springs.

Given data, $P = 25 kW$

$N = 3000$

$\frac{r_1}{r_2} = 1.25 \Rightarrow r_1 = 1.25 r_2$

$\mu = 0.255$

$P = 2\pi N T/60$

$\Rightarrow T = \dfrac{P \times 60}{2\pi N} = 79.5$ N-m

$T = n W \mu R$

$\Rightarrow R = \dfrac{T}{n W \mu}$

for uniform press.

$P \times r_2 = C$

$\Rightarrow 0.1 \times r_2 = C \Rightarrow C = 0.1 r_2$

$W = 2\pi C (r_1 - r_2)$

$= 2\pi\, 0.1 r_2 (r_1 - r_2)$

$= 2\pi\, 0.1 r_2 \cdot r_2 (r_1/r_2 - 1)$

$= 2\pi \times 0.1 r_2^2 (1.25 - 1)$

$= 2\pi \times 0.1 r_2^2 \times 0.25$

$W = 0.157 r_2^2$

$T = n \mu W R$ $\quad$ ($\because n = 2$ bcz both side effective)

$= 2 \times 0.255 \times 0.157 r_2^2 \times \left(\dfrac{r_1 + r_2}{2}\right)$

$= 0.08007 \times r_2^2 \times \left(\dfrac{1.25 r_2 + r_2}{2}\right)$

$79.5\,m = 0.08007 \times r_2^2 \times \dfrac{2.25 r_2}{2}$

$79.5 \times 10^3\,mm = 0.08007 \times r_2^2 \times 1.125 r_2$

$\cancel{\dfrac{79.5}{25 \times 10^3}}$

$79.5 \times 10^3 = 0.09007\cancel{4}75\, r_2^3$

$\Rightarrow r_2^3 = \dfrac{\cancel{25 \times 10^3}\ 79.5 \times 10^3}{0.09007875}$

$r_2^3 = \cancel{277.53 \times 10^3}$

$\Rightarrow r_2 = \sqrt[3]{277.53 \times 10^3}$

$\Rightarrow r_2^3 = 882.56$

$\Rightarrow r_2 = \sqrt[3]{882.56} = 9.59\ m = 96\ mm$

$r_2^3 = \dfrac{79.5 \times 10^3}{0.09007875}$

$r_2 = \sqrt[3]{882561.09}$

$r_2 = 96\ mm$

$r_1 = 1.25 \times 96 = 120\ mm$

$r_1 = 1.25 \times r_2 = 1.25 \times 9.59 = 12\ m = 1200\ mm$

$W = 0.157 \times r_2^2$

$= 0.157 \times 96^2$

$= 1446.9 \simeq 1447\ N$

$= \cancel{0.157 \times 9.59^2}\ 9$

$= 14.43\ kN$

$= 0.157 \times (96)^2 = 1447\ N$

3. A single dry plate clutch transmit 7.5 kw at 900 rpm. The axial pren. is limited to 0.07 N/mm². If the co-efficient of friction is 0.25, find

1. Mean radius & face width of the friction lining assuming the ratio of the mean radius to the face width as 4.

11. Outer & inner radii of the clutch plate.

Given data, $P = 7.5\ kW$

$N = 900\ rpm$

$p = 0.07\ N/mm^2$

$\mu = 0.25$

$$\frac{R}{(r_1-r_2)} = 4$$

$$\frac{\frac{r_1+r_2}{2}}{r_1-r_2} = 4$$

$$\frac{r_1+r_2}{r_1-r_2} = 8 \qquad \frac{R}{w} = 4$$

$$r_1+r_2 = 8r_1 - 8r_2 \qquad \Rightarrow \frac{R}{4} = w$$

$$9r_2 = 7r_1$$

$$\Rightarrow r_1 = \frac{9}{7}r_2 \qquad \omega =$$

$$P = \frac{2\pi N T}{60}$$

$$\Rightarrow T = \frac{60P}{2\pi N} = \frac{60 \times 7.5\times10^3}{2\times\pi\times900} = \frac{\cancel{Watt}\ N\ m/sec}{r.p.m}$$

$$\cancel{= 8382\ N\text{-}m} \qquad Nm$$

$$= 79.57\ N\text{-}m$$

$$A = 2\pi R w \qquad = 79.57\times10^3\ Nmm$$

$$W = A \times P$$

$$= 2\pi R w \times P$$

$$= 2\pi R \times \frac{R}{4} \times 0.07$$

$$= 0.11 R^2$$

$$T = n\mu W R$$

$$= 2\times 0.25 \times 0.11R^2 \times R$$

$$79.57\times10^3 = 0.055R^3$$

$$\Rightarrow R^3 = 1.446\times10^6$$

$$\Rightarrow R = 113.099$$

$$R = \frac{r_1 + r_2}{2} = 113\,mm$$

$\Rightarrow r_1 + r_2 = 226\,mm$ — (I)

$w = \frac{R}{4} = \frac{113}{4} = 28.25 = r_1 - r_2$

$\Rightarrow r_1 - r_2 = 28.25$ — (II)

eqn (I) − eqn (II)

$\cancel{r_1} + r_2 - (\cancel{r_1} - r_2) = 197.75$

$2r_2 = 197.75$

$\Rightarrow r_2 = 98.875$

$r_1 - r_2 = 28.25\,mm \Rightarrow r_1 = 127.125\,mm$

4. A dry single plate clutch is to be designed for an automative velchle whose engine is rated to give 100 kW at 2400 rpm & the max. torque 500 N-m. The outer radius of the friction plate is 25% more than the inner radius. the intensity of pres. betn the plate is not to exceed 0.07 N/mm². The co-efficient of friction may be assumed as 0.3. The helical springs required by this clutch to provide axial force necessary to engage the clutch are 8. If each springs has stiffness equal to 40 N/mm, ~~the ci~~ determine the initial comp. of the spring & dimensions of the friction plate.

~~$P = 100\,kW$~~

$N = 2400\,rpm$ $\quad\quad \mu = 0.3$

$T = 500\,N\text{-}m$

$r_1 = r_2 + \cancel{0.25\%}\; 0.25 r_2$

$r_1 = 1.25 r_2$

$P = 0.07\,N/mm^2$

$\mu = 0.3$

$P = \frac{2\pi N T}{60}$

$\Rightarrow T = \frac{60P}{2\pi N} = \frac{60 \times}{}$

$P \times r_2 = C$

$\Rightarrow C = 0.07 r_2$

$W = \mu \omega R$

$W = 2\pi C (r_1 - r_2)$

$= 2\pi \times 0.07 r_2 (1.25 r_2 - r_2)$

$= 2\pi \times 0.07 r_2 \cdot r_2 (1.25 - 1)$

$= 2\pi \times 0.07 r_2^2 \cdot 0.25$

$= 0.11 r_2^2$

$R = \frac{r_1 + r_2}{2}$

$= \frac{1.25 r_2 + r_2}{2}$

$= \frac{2.25 r_2}{2}$

$= 1.125 r_2$

$T = n \mu W R$

$= 2 \times 0.3 \times 0.11 r_2^2 \times 1.125 r_2$

$500 \times 10^3 = 0.07425 r_2^3$

$\Rightarrow r_2^3 = \frac{500 \times 10^3}{0.07425} = 6.73 \times 10^6$

$\Rightarrow r_2 = 188.8$ mm

$r_1 = 1.25 r_2 = 1.25 \times 188.8 = 236$ mm

Initial comp. $= \frac{W}{\text{total stiffness}} = \frac{0.11 \times 188.8^2}{8 \times 40}$

$= 12.25$

total stiffness,

$S = 40 \times 8$

$= 320$

Power transmissions in belt drives:

Introduction:

→ The belts are used to transmit power from one shaft to another by means of pulley.

→ The amount of power transmissions depends upon the following factors,

(1) velocity of the belt

(2) The tension under which the belt is placed on the pulley.

(3) The arc of contact betn the belt & the smaller pulley.

(4) The condition under which the belt is used.

Velocity ratio of the belt drive :-

It is the ratio betn the velocities of the driver & the follower.

Let, d_1 = diameter of the driver

d_2 = diameter of the follower.

N_1 = Speed of the driver in rpm

N_2 = Speed of the follower in rpm

Length of the belt that passes over the driver in 1 min. $= \pi d_1 N_1$

Length of the belt, passes over the follower in 1 min. $= \pi d_2 N_2$

Since the length of the belt passes over the driver in one min. is equal to the length of the belt passes over the follower in one min.

Therefore, $\pi d_1 N_1 = \pi d_2 N_2$

Or velocity ratio, $\cancel{\frac{N_1}{N_2}} = \cancel{\frac{d_2}{d_1}}$ $\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$

when thickness of the belt is consider, then velocity is equal to t, $\boxed{\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}}$

Velocity ratio of a compound belt drive:-

let, d_1 = diameter of the pulley-1

N_1 = Speed of the pulley-1

d_2, d_3, d_4 & N_2, N_3, N_4 = corresponding values of Pulley-2, -3, -4.

We know that velocity ratio of pulley-1 & 2.

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{---(i)}$$

Similarly V.R of pulley-3 & 4.

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \quad \text{---(ii)}$$

Multiply eqn (i) & (ii), we get

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\frac{\cancel{N_2}}{N_1} \times \frac{N_4}{\cancel{N_3}} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$ (∵ As, $N_2 = N_3$ being keyed to the same shaft)

$$\frac{N_4}{N_1} = \frac{d_1 d_3}{d_2 d_4}$$

It there are six no. of pulleys therefore

$$\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}$$

Speed of last driven devided by speed of first driver equel to the product of the diameters of the driven devided by product of the diameter of the follower

Slip of a belt :-

The belt & the shafts are having contact with a firm frictional grip. But sometimes the frictional grip is insufficient. This may cause some forward motion of the driver without carrying the belt. This may also cause some forward motion of the belt without carrying the driven pulley. This phenomenon is called as slip of the belt.

It is generally expressed as percentage

Let, S_1 % = slip betn the driver & the belt

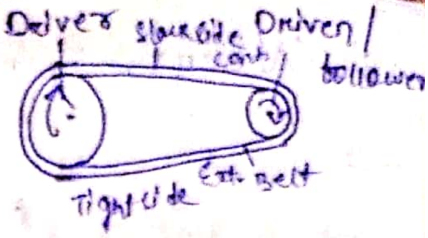
S_2 % = slip betn the belt & the follower.

Velocity of the belt passing over the driver per second ; $v = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100}$

$$v = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right) \quad \text{—(i)}$$

velocity of the belt passing over the follower

Per second, $v = \frac{\pi d_2 N_2}{60}$



$$\frac{\pi d_2 N_2}{60} = v - v \times \frac{S_2}{100}$$

$$= v\left(1 - \frac{S_2}{100}\right) = \frac{\pi d_1 N_1}{60}\left[1 - \frac{S_1}{100}\right]\left[1 - \frac{S_2}{100}\right]$$

~~Put the value of v in above eqn we get,~~ $\because$ neglecting $\frac{\pi d_1 N_1}{60} \times \frac{S_1}{100} \times \frac{S_2}{100}$

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left[1 - \frac{S_1}{100} - \frac{S_2}{100}\right]$$

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1 + S_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right)$$

Where, $S = S_1 + S_2$ i.e. total percentage of sleep.

When belt thickness is considered

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{S}{100}\right)$$

Creep of the belt :-

When the belt passes from slack side to tight, a certain purtion of the belt extents & it contracts again when the belt passes from tight side to slack side.

Due to these changes in length, there is a relative motion betn the belt & the pulley. This relative motion is known as Creep.

Considering creep, the V.R. is given by,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

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Where, σ_1 & σ_2 = Stress on the belt on the tight side & slack side respectively.

E_1 = Young's modulus for the material of the belt.

Question :-

1. An engine running at 150 rpm drives a line shaft by means of a belt. The engine pulley is 750 mm diameter & the pulley on the line shaft being 450mm. A 900 mm pulley on the line shaft drives a 150 mm diameter pulley kid to a dynamo shaft. Find the speed of the dynamo shaft when,

i. There is no slip

ii. There is a slip of 2% at each drive

$N_1 = 150$ rpm

$d_1 = 750$ mm

$d_2 = 450$ mm

$d_3 = 900$ mm

$d_4 = 150$ mm

$$\frac{N_1}{N_4} = \frac{d_2 \times d_4}{d_1 \times d_3}$$

$$\Rightarrow N_4 = \frac{N_1}{\frac{d_2 \times d_4}{d_1 \times d_3}} = \frac{150}{\frac{450 \times 150}{750 \times 900}} = 1500$$

2. The power is transmitted from a pulley 1m. diameter running at 200 rpm to a pulley 2.25 m diameter by means of a belt. Find the speed lost by the driven pulley as a result of creep, if the stress on the tight & slake sides 1.4Mpa & 0.5 Mpa respectively. The youngs modulus for the material of belt is 100 Mpa.

$d_1 = 1m.$ $\quad N_1 = 200\, rpm$

$d_2 = 2.25$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

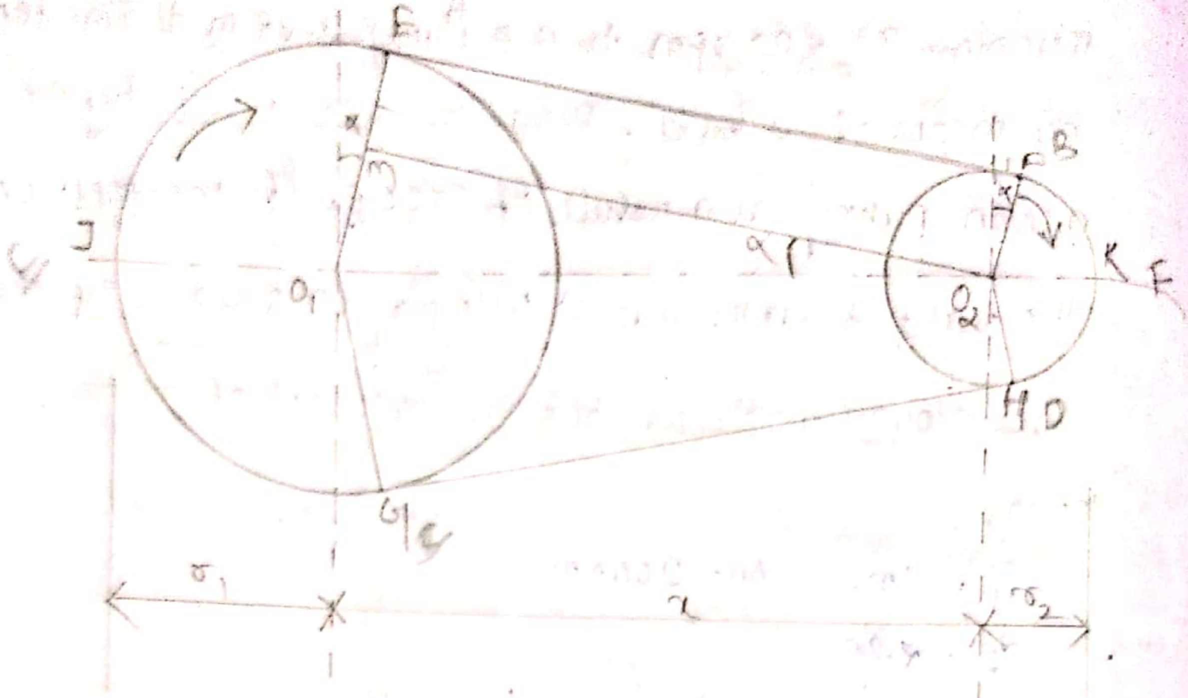
$$\Rightarrow N_2 = \frac{1}{2.25} \times 200 = 88.88 \text{ rpm}$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

$$N_2 = \frac{d_1}{d_2} \times N_1 \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

$$= \frac{1}{2.25} \times 200 \times \frac{100 \times 10^6 + \sqrt{0.5^2}}{100 \times 10^6 + \sqrt{1.4^2}}$$

Length of an open belt drive :-



Let r_1 & r_2 is radii of the larger & smaller pulley.

x = Dist. betn the centre of two pulleys $(O_1 O_2)$

L = Total length of the belt

The belt leaves the larger pulley at E & G & the smaller pulley at F & H. Through O_2 draw $O_2 m$ parallel to F E.

From the geometry of the figure we find that $O_2 m$ is $\perp^r$ to $O_1 E$.

Let $m O_2 O_1 = \alpha$ radian

Now length of the belt,

L = Arc GJE + EF + Arc FKH + HG

$= 2$ (Arc JE + EF + Arc FK) —— (1)

From the geometry of the figure, ~~sin α~~

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2}$$

$$= \frac{r_1 - r_2}{x}$$

Since α is very small,

$$\sin\alpha = \alpha = \frac{r_1 - r_2}{x} \quad \text{—— (ii)}$$

Arc $JE = r_1 \times \left(\frac{\pi}{2} + \alpha\right)$ —— (iii)

Arc $FK = r_2 \times \left(\frac{\pi}{2} - \alpha\right)$ —— (iv)

(Score $= r\,\theta$ (radian)
$S = \frac{\theta}{360} \times 2\pi r$ (degree))

$$EF = MO_2 = \sqrt{O_1O_2^2 - O_1M^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

Expanding the above eqn by binomial theorem,

$$EF = x\left(1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \ldots\ldots\right)$$

$$EF = x - \frac{(r_1 - r_2)^2}{2x} \quad \text{—— (v)}$$

Substituting the value of eqn (3), (4) & (5) in eqn (1),

$$L = 2\left(r_1 \times \left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \left(\frac{\pi}{2} - \alpha\right)\right)$$

$$= 2\left(r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2\frac{\pi}{2} - r_2\alpha\right)$$

$$= 2\left(\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right)$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + \cancel{2x} - \frac{(r_1 - r_2)^2}{x}$$

Substitute the value of 'x' from eqn-(ii)

$$L = \pi(r_1+r_2) + \frac{2(r_1-r_2)^2}{x} + 2x - \frac{(r_1-r_2)^2}{x}$$

$$\boxed{L = \pi(r_1+r_2) + \frac{(r_1-r_2)^2}{x} + 2x}$$ (In terms of pulley radii)

$$\boxed{L = \frac{\pi}{2}(d_1+d_2) + \frac{(d_1-d_2)^2}{4x} + 2x}$$ (In terms of Pulley dia.)

Belt :-

→ Belt are used to transmit power from one shaft to another with the help of pulley.

→ The other methods of transmitting powers are rope, chain.

→ Belt, rope is used ~~where~~ where the distance betn the shaft is large.

→ Gears are used where the distance is small.

Types of ~~belt~~ belt drive :-

It is classified into 3 types.

1. Light belt drive
2. Medium belt drive
3. Heavy belt drive

1. Light belt drive :-

These are used to transmit small powers at belt speed upto about 10m/s as in agricultural machines & small machine tools.

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2. Medium belt drive :-

These are used to transmit ~~po~~ medium power at belt speeds over 10 m/s ~~but~~ to ~~speed~~ 22 m/s as in machine tools.

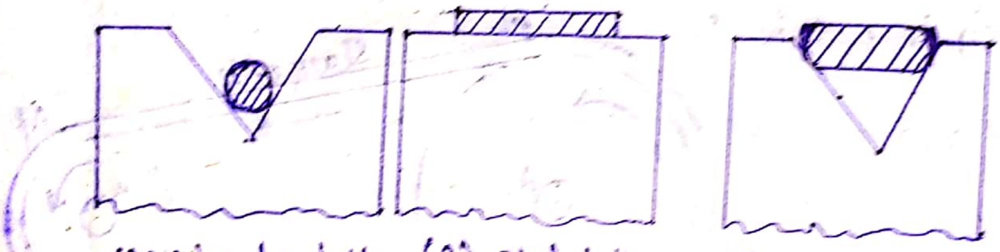
3. Heavy belt drive :-

These are used to transmit large powers at belt speeds above 22 m/s. as in ~~as~~ compressors & generators.

Types of belts :-

It is of three types.

~~1. Flat~~ belt :



(c) circular belt (a) Flat belt (b) V-belt

1. Flat belt :-

→ It is rectangular in cross section.

→ It is mostly used in factories & workshops, where moderated amount of power is to be transmitted, from one pulley to another pulley, when they are not more than 8 metres apart.

2. V-belt :-

→ It is trapizoidal in cross section.

→ It is mostly used in factories & workshops where moderate amount of power is to be transmitted, from one pulley to another pulley, when they are near to each other.

3. ~~Circular~~.

3. Circular belt or rope :-

→ It is circular in cross-section.

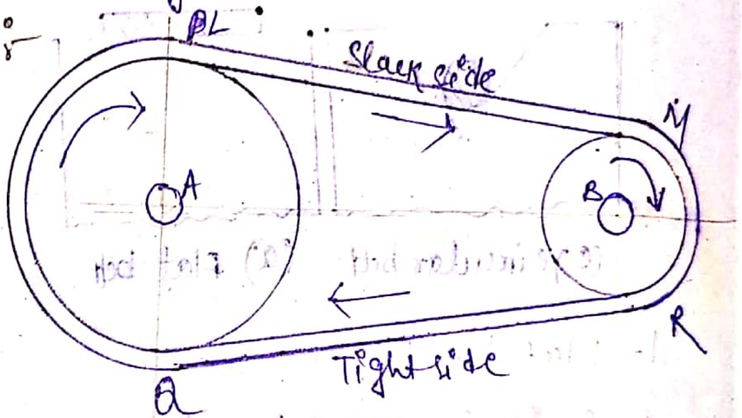
→ It is mostly used in factories & workshops, where a great amount of power is to be transmitted, from one pulley to another, when they are more than 8 metres apart.

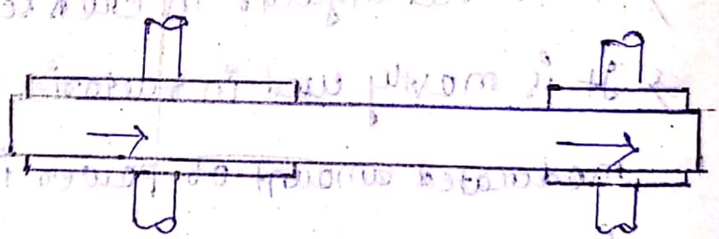
Materials of belt :- Leather, cotton or fabric, rubber

Types of flat belt drive :-

It is classified into following types,

1. Open belt drive :-



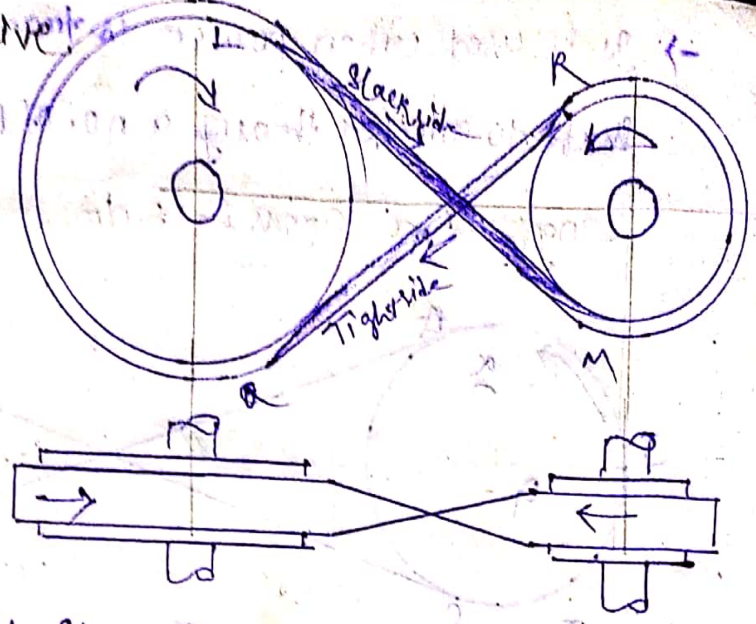


→ The open belt drive is used with the shaft arranged in parallel and rotating in same direction.

→ In this case, the driver pulls the belt from one side & delivers it to other side. Thus tension in the lower side belt is more than ~~the up~~ the upper side belt.

→ Therefore the lower side belt is called tight side & upper side is called slack side.

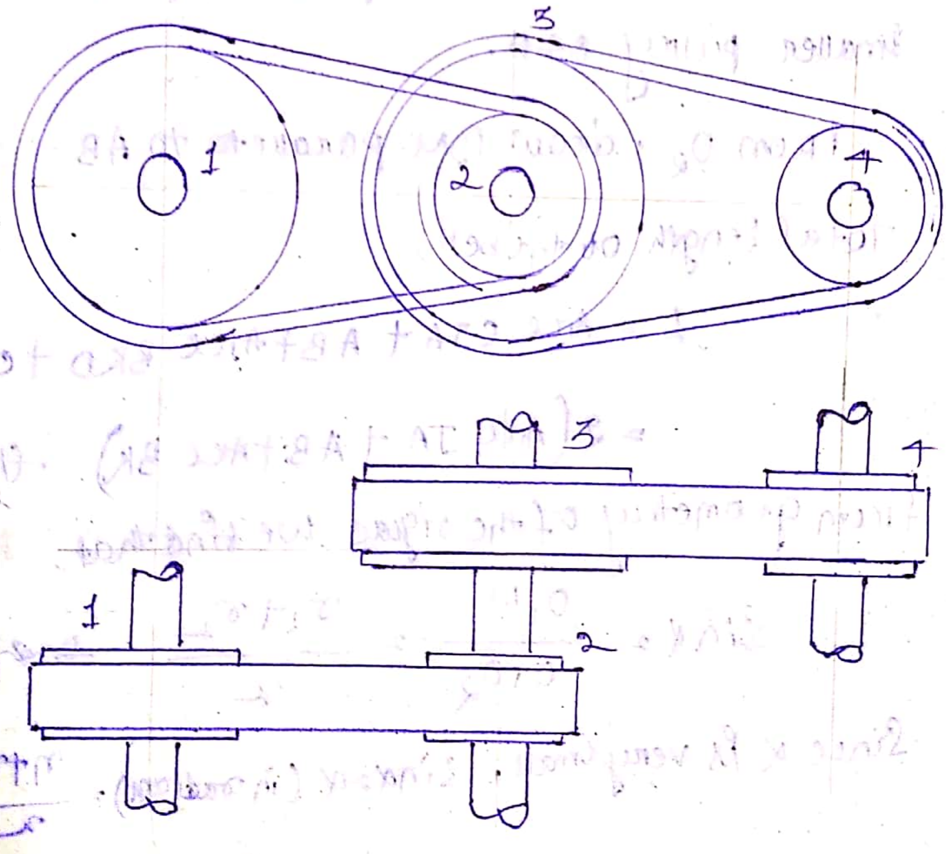
2. Cross belt drive:



→ It is used with shafts arranged parallel & rotating in opposite directions.

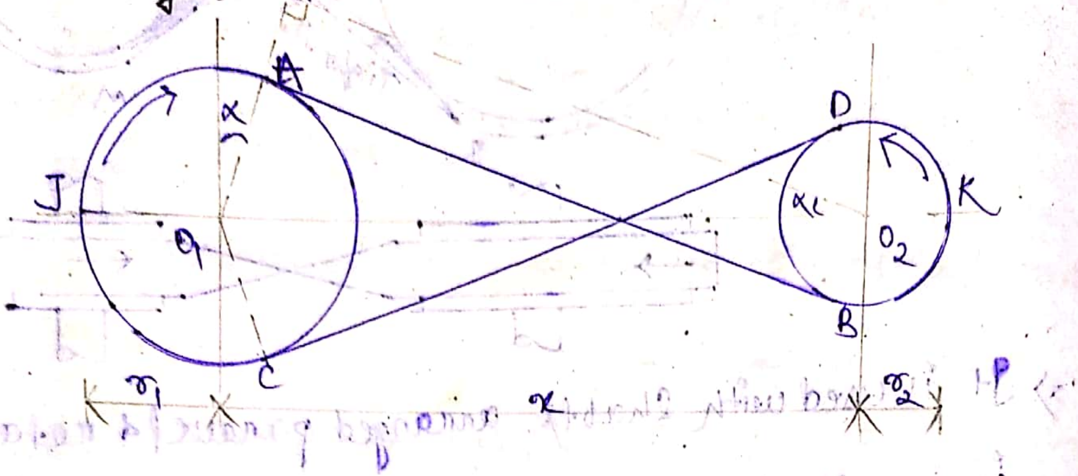
→ A little consideration will show that at a point where the belt crosses, it rubs against each other & there will be excessive wear & tear. In order to avoid this the shaft will place at a max. distance.

3. Compound belt drive.



→ It is used when power is transmitted from one shaft to another through a no. of pulleys.

Length of a cross belt drive :-



Consider a cross belt drive.

r_1 & r_2 = radii of the larger & smaller pulleys

x = Distance betn the centre of two pulleys

L = Total length of the belt.

Let the belt leaves the larger pulley at A & C & the smaller pulley B & D.

From O_2, draw O_2M parallel to AB.

Total length of the belt,

$$L = \text{Arc } CJA + AB + \text{Arc } BKD + CD$$

$$= 2(\text{Arc } JA + AB + \text{Arc } BK) \quad -(i)$$

From geometry of the figure we find that,

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{r_1+r_2}{x} \quad -(ii)$$

Since α is very small, $\sin\alpha = \alpha$ (in radian) $= \frac{r_1+r_2}{x}$ -(iii)

Arc $JA = \left[\frac{\pi}{2} + \alpha\right] \times r_1$ —(III)

Arc $BK = \left[\frac{\pi}{2} + \alpha\right] \times r_2$ —(IV)

$AB = MO_2 = \sqrt{O_1O_2^2 - O_1M^2} = \sqrt{x^2 - (r_1+r_2)^2}$

$$= x\sqrt{1 - \left(\frac{r_1+r_2}{x}\right)^2}$$

Expanding this eqn by binomial theorem,

$$AB = x\left[1 - \frac{1}{2}\left(\frac{r_1+r_2}{x}\right)^2 + \dots\right]$$

$$\therefore AB = x - \frac{(r_1+r_2)^2}{2x} \quad \text{—(V)}$$

Substituting the value of eqn ~~(II)~~, (III), (IV), (V) in eqn (I) we get,

$$L = 2\left[\left(\frac{\pi}{2}+\alpha\right)r_1 + x - \frac{(r_1+r_2)^2}{2x} + \left(\frac{\pi}{2}+\alpha\right)r_2\right]$$

$$= 2\left[r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1+r_2)^2}{2x} + r_2\frac{\pi}{2} + r_2\alpha\right]$$

$$= r_1\pi + 2r_1\alpha + 2x - \frac{(r_1+r_2)^2}{x} + r_2\pi + 2r_2\alpha$$

$$= \pi(r_1+r_2) + 2\alpha(r_1+r_2) + 2x - \frac{(r_1+r_2)^2}{x}$$

Put the value of eqn (II), (III) in the above eqn.

$$= \pi(r_1+r_2) + \frac{2(r_1+r_2)}{x}(r_1+r_2) + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + 2x + \frac{2(r_1+r_2)^2}{x} - \frac{(r_1+r_2)^2}{x}$$

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}$$

Governor :- Chapter-1

The function of a governor is to regulate the meanspeed of an engine, when there is are variation in the load.

~~Ex:~~

Types of governor

- Governor
  - Centrifugal governor
    - Pendulum type
      - Watt governor
    - Loaded type
      - Dead weight governors
        - Porter governor
        - Proell governor
      - Spring controlled governor
        - Hartnell governor
        - Hartung governor
        - Wilson-hartnell governor
        - Pickering governor
  - Inertia gover

Watt Governor :-

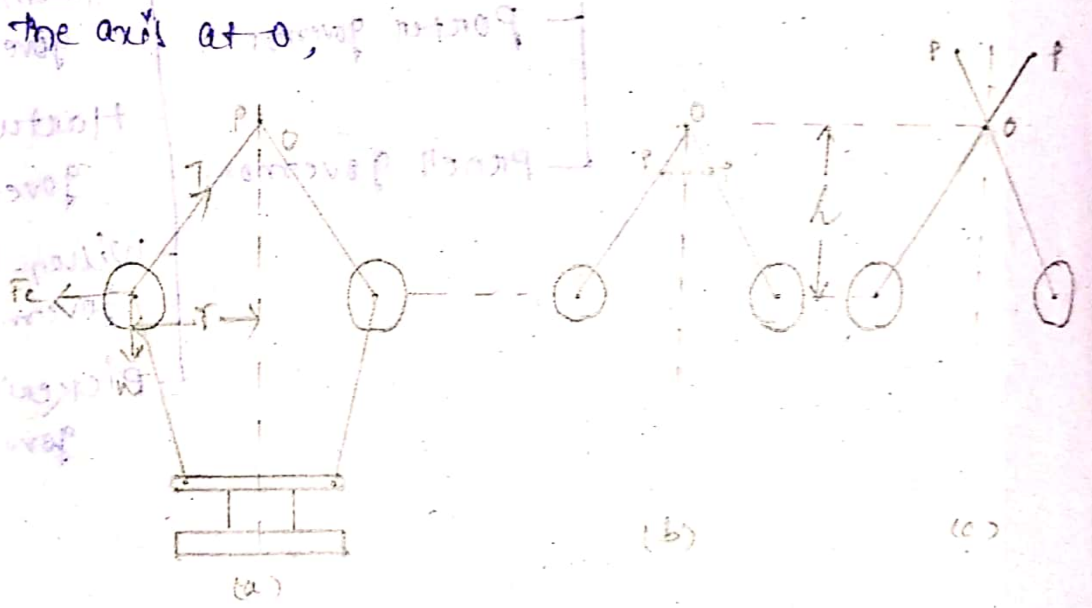
→ The simplest form of a Centrifugal governor is a watt governor. It is basically a conical pendulum with links attached to a sleeve of negligible mass.

→ The arms of the governor maybe connected to the spindle in the following three ways.

① The pivot P, may be on the spindle axis.

② The pivot P, maybe offset from the spindle axis & the arms when produced intersect at O,

③ The pivot P, may be offset, but the arms cross the axis at O,



m = mass of the ball in kg

w = weight of the ball in newtons = mg

T = Tension in the arm in newtons.

ω = Angular vel. of the arm and ball about the spindle axis in rad/s.

r = Radius of the path of rotation of the ball

Fc = Centrifugal force acting on the ball in newtons $= m\omega^2 r$

h: Height of the governor in meters

It is assumed that the weight of the arms, links, the sleeve are negligible as compared to the weight of the ball. Now, the ball is in equilibrium under the action of

i) The centrifugal force (F_c) acting on the ball

ii) The tension in the arm

iii) The weight of the ball.

Taking moments about point O, we have

$$F_c \times h = W \times r = mgr$$

$$F_c \times h = mgr$$

$$m\omega^2 \quad \cancel{m} \times \omega^2 \times \cancel{r} \times h = \cancel{m} g \cancel{r}$$

$$h = g/\omega^2$$

When g is expressed in m/s^2 & ω in rad/s then h in metres.

If N is the speed in r.p.m,

$$\text{then } \omega = \frac{2\pi N}{60}$$

$$\therefore h = \frac{9.81}{\left(\frac{2\pi N}{60}\right)^2} = \frac{9.81 \times \left(\frac{60}{2\pi}\right)^2}{N^2}$$

$$\cancel{895} = \frac{895}{N^2} \, m.$$

$$\therefore \boxed{h = \frac{895}{N^2}}$$

Problem :-

$h = ?$

$N_1 = 60$ rpm

$N_2 = 61$ rpm

$dh = ?$

$$h_1 = \frac{895}{60^2} = 0.248 \text{ m}$$

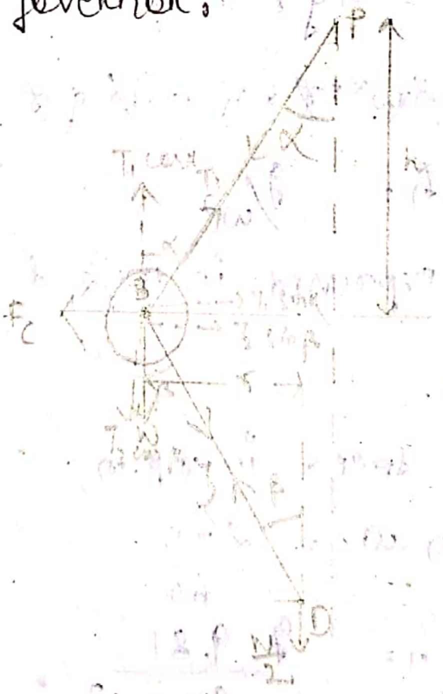
$$h_2 = \frac{895}{61^2} = 0.24 \text{ m}$$

Change in vertical height,

$$dh = h_1 - h_2 = 0.248 - 0.24$$
$$= 0.008 \text{ m}$$
$$= 8 \text{ mm}$$

Porter governor:-

Porter- [illegible]



The porter governor is a modification of watt's governor, with central load attached to the sleeve. The load moves up & down the central spindle.

m = mass of each ball in Kg
w = weight " " " Newton
M = Mass of central load in Kg
W = weight " " " in newton.
r = radius of rotation in meters.
h = Height of governor in metres
N = speed of the ball in rpm
ω = Ang. speed of the ball in rad/s
F_c = centrifugal force acting on the ball.

α = Angle of inclination of the arm to the vertical
β = ~~Ang~~ " " " " " link " " "

Method of Resolution of forces:

Considering eq at D

$$\underline{T_2 \cos\beta = \frac{W}{2} = \frac{Mg}{2}} \quad \cancel{T_1 \cos\alpha = T_2}$$

$$T_2 \cos\beta = \frac{W}{2}$$

$$T_2 = \frac{Mg}{2\cos\beta} \qquad T_2 = \frac{Mg}{2\cos\beta}$$

Resolving forces vertically at point B.

$\cancel{T_1 = w}$.

$$T_1 \cos\alpha = T_2 \cos\beta + w$$

$$= T_2 \cos\beta + mg \quad \text{--- (ii)}$$

$$= \frac{Mg}{2} + mg$$

$$T_1 \sin\alpha + T_2 \sin\beta = F_c$$

$$T_1 \sin\alpha + \frac{mg}{2\cos\beta} \sin\beta = F_c$$

$$T_1 \sin\alpha = F_c - \frac{mg}{2} \tan\beta \quad \text{(iii)}$$

Deviding eqn-(iii) by eqn (ii) we get,

$$\frac{\cancel{T_1} \sin\alpha}{\cancel{T_1} \cos\alpha} = \frac{F_c - \frac{mg}{2} \tan\beta}{\frac{Mg}{2} + mg}$$

$$\left(\frac{Mg}{2} + mg\right) \tan\alpha = F_c - \frac{mg}{2} \tan\beta$$

$$\frac{Mg}{2} + mg = \frac{F_c}{\tan\alpha} - \frac{mg}{2} \frac{\tan\beta}{\tan\alpha}$$

let $\frac{\tan\beta}{\tan\alpha} = q$

$\tan\alpha = \frac{r}{h}$

Then, $\frac{mg}{2} + mg = m\omega^2 \times \cancel{r} \times \frac{h}{\cancel{r}} - \frac{mg}{2} \times q$

$$\frac{Mg}{2}(1+q) + mg = m\omega^2 h$$

$$\Rightarrow h = \left[\frac{Mg}{2}(1+q) + mg\right] \times \frac{1}{m\omega^2}$$

$$\Rightarrow \omega^2 = \left[\frac{Mg}{2}(1+q) + mg\right] \times \frac{1}{m \times h}$$

$$\Rightarrow \left(\frac{2\pi N}{60}\right)^2 = \left[\frac{Mg}{2}(1+q) + mg\right] \times \frac{1}{mh}$$

$\Rightarrow N^2 = \left[\frac{Mg}{2}(1+q) + mg\right] \times 895$

$\Rightarrow \boxed{N^2 = \left[\frac{M}{2}(1+q) + m\right] \times \frac{g}{mg} \times \frac{895}{h}}$

(Taking $g = 9.81$)

Case-1

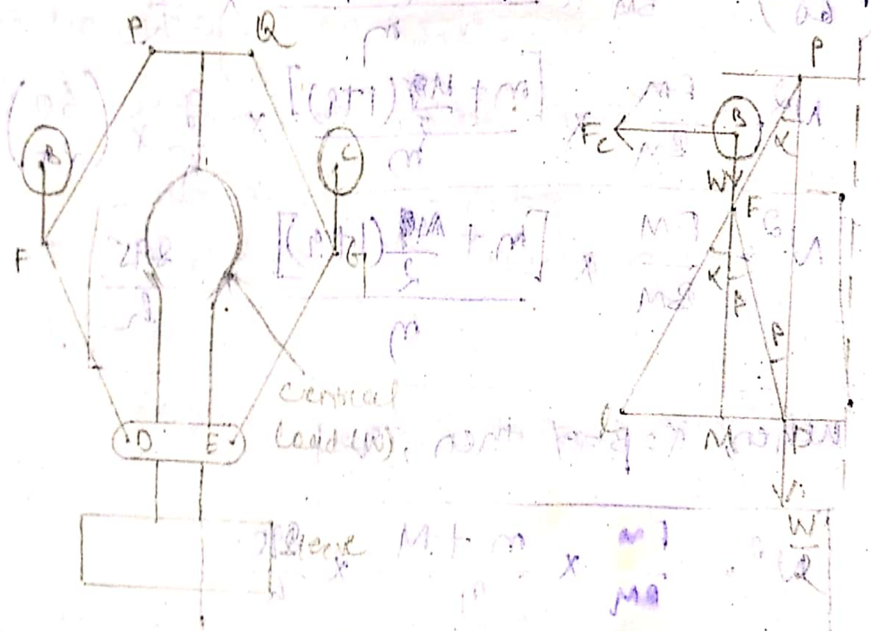
when, $\tan\alpha = \tan\beta$ or $q = 1$.

$\therefore \boxed{N^2 = \frac{M+m}{m} \times \frac{895}{h}}$

Proell Governor :-

The proell governor has the balls fixed at B & C to the extension of the links DF & EG.

The arms FP & GQ are pivoted at P & Q respectively



Taking moments about I, using the same notations

$F_c \times BM = w \times IM + \frac{W}{2} \times ID = mg \times Im + \frac{Mg}{2} \times ID$

$\frac{F_c \times BM}{FM} = mg \times \frac{Im}{FM} + \frac{Mg}{2} \times \left(\frac{IM + MD}{FM}\right)$

$$\frac{F_C \times BM}{FM} = mg \times \tan\alpha + \frac{Mg}{2} \times (\tan\alpha + \tan\beta)$$

$$\frac{F_C \times BM}{FM} = \tan\alpha\left(mg + \frac{Mg}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right)$$

Let, $\frac{\tan\beta}{\tan\alpha} = q$ & $\tan\alpha = \frac{r}{h}$

Then,

$$m.\omega^2 r \times \frac{BM}{FM} = \frac{r}{h}\left(mg + \frac{Mg}{2}(1+q)\right)$$

$$\omega^2 = \frac{FM}{BM} \frac{\left[m + \frac{M}{2}(1+q)\right]}{m} \times \frac{g}{h}$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{FM}{BM} \times \frac{\left[m + \frac{M}{2}(1+q)\right]}{m} \times \frac{g}{h}$$

$$N^2 = \frac{FM}{BM} \times \frac{\left[m + \frac{M}{2}(1+q)\right]}{m} \times \frac{g}{h} \times \left(\frac{60}{2\pi}\right)^2$$

$$\boxed{N^2 = \frac{FM}{BM} \times \frac{\left[m + \frac{M}{2}(1+q)\right]}{m} \times \frac{895}{h}}$$

when, $\alpha = \beta$ then, $q = 1$

$$\boxed{N^2 = \frac{FM}{BM} \times \frac{m + M}{m} \times \frac{895}{h}}$$

Hartnell governor:-

- A hartnell governor is a spring loaded governor.
- It consists of bell crank levers pivoted at the points O,O to the frame.
- The frame is attached to the governor spindle & therefore rotates with it.
- Each lever carries a ball at the end of vertical arm OB & a roller at the horizontal arm OR.
- A helical spring in compression provides equal downward forces on the two roller through a collar on the sleeve.
- The spring force may be adjusted by screwing a nut up or down on sleeve.

m = Mass of each ball in kg

M = Mass of sleeve in kg

r_1 = Min. radius of rotation in metres.

r_2 = Max. " " " " ".

ω_1 = Angular speed of the governor at min. radius in rad/s.

ω_2 = " " " " " " max. radius in rad/s.

S_1 = Spring force exerted on the sleeve at ω_1 in N,

S_2 = " " " " " ω_2 in N.

F_{c1} : centrifugal force at ω_1 in N.

F_{c2} = " " " ω_2 " ".

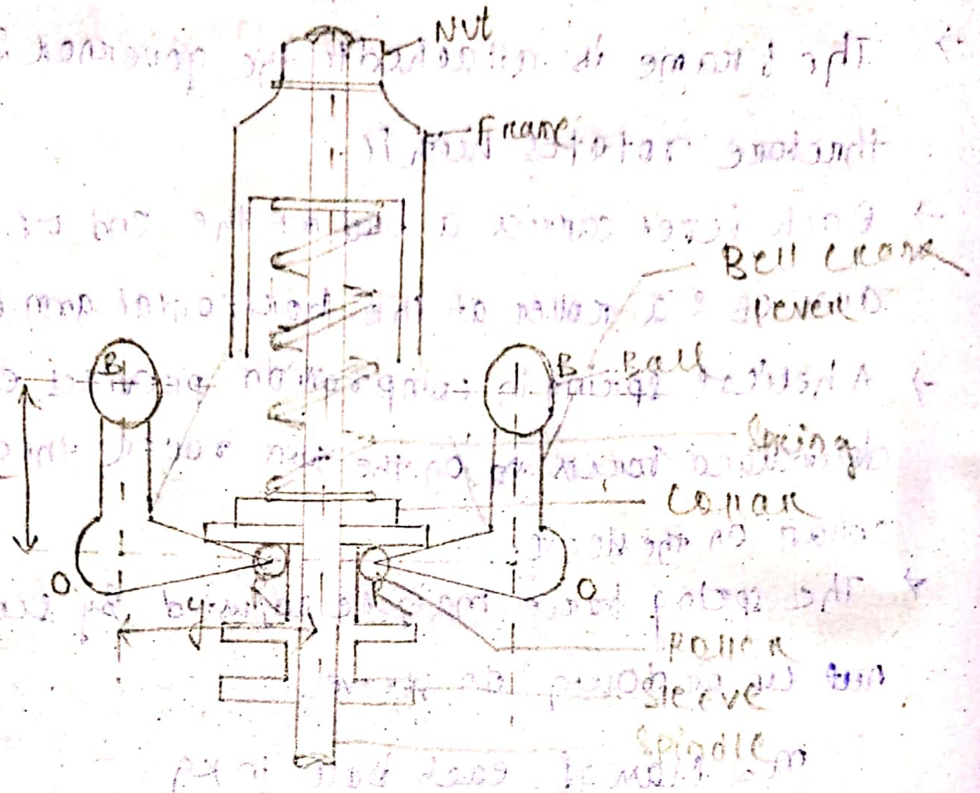
S = stiffness of the spring

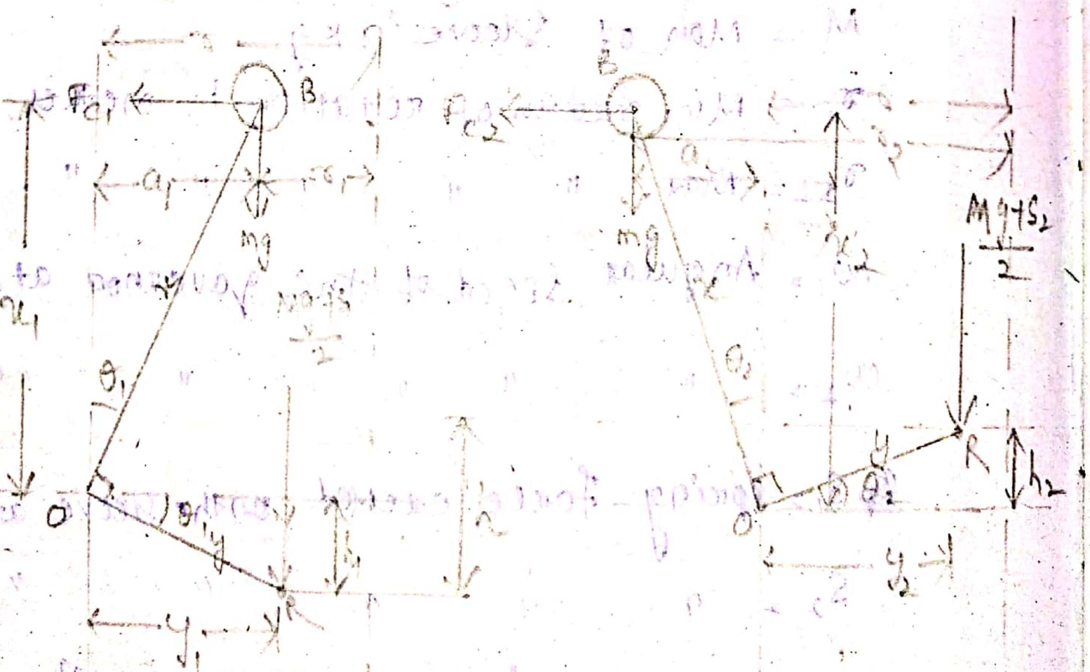
~~x =~~ length.

x = length of the vertical or ball arm of the lever in m.

y = " " horizontal or sleeve arm " "

r = Distance of fulcrum 'O' from the governor axis or the rad. of rotation when the governor in mid position.





For the min. position,

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \quad -(\text{I})$$

For the max. position,

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \quad -(\text{II})$$

Addying eqn -(I) & (II) we get,

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x}$$

$$\frac{h}{y} = \frac{r_2 - r_1}{x}$$

$$\Rightarrow h = (r_2 - r_1) \times \frac{y}{x} \quad -(\text{III})$$

For min. position Taking moment about 'O'.

$$\cancel{F_c \times x_1 =}$$

$$\frac{Mg + S_1}{2} \times y_1 = F_c \times x_1 - mg \times a_1 \quad -(\text{IV})$$

For max. position Taking moment about 'O'.

$$\frac{Mg + S_2}{2} \times y_2 = F_c \times x_2 - mg \times a_2 \quad -(\text{V})$$

let, $x_1 = x_2 = x$

$y_1 = y_2 = y$

$mg \simeq 0$

$$\frac{Mg + S_1}{2} \times y_1 = F_{c_1} x_1$$

$$\Rightarrow Mg + S_1 = \frac{2 F_{c_1} x_1}{y_1} = 2 F_{c_1} \frac{x}{y} \quad -(\text{VI})$$

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$$\frac{Mg + S_2}{2} \times y_2 = F_{c2}\, x_2$$

$$Mg + S_2 = \cancel{F_c}\, 2 \times F_{c2} \times \frac{x}{y} \quad -(vii)$$

Subtracting eqn -(vi) from eqn -(vii) we get,

$$S_2 - S_1 = 2 \times (F_{c2} - F_{c1}) \times \frac{x}{y}$$

We know $\underline{S_2 - S_1 = hs}$ $\quad$ & $h = (r_2 - r_1) \times \frac{y}{x}$

$$\Rightarrow S = \frac{S_2 - S_1}{h} = \frac{2 \times (F_{c2} - F_{c1}) \times \frac{x}{y}}{(r_2 - r_1) \times \frac{y}{x}}$$

$$\boxed{S = 2\left(\frac{F_{c2} - F_{c1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2}$$

Flywheel :-

→ A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

→ A flywheel controls the speed variations caused by fluctuation of the engine turning moment during each cycle of operation.

Co-efficient of fluctuation of speed :-

→ The difference betn max. & min. speeds during a cycle is called Maximum fluctuation of speed.

→ The ratio of maximum fluctuation of speed to the mean speed is called the co-efficient of fluctuation of speed.

N_1 & N_2 = ~~min~~ Max. & min. speeds in rpm during the cycle, and

N = Mean speed in rpm $= \dfrac{N_1+N_2}{2}$

∴ co-efficient of fluctuation of speed,

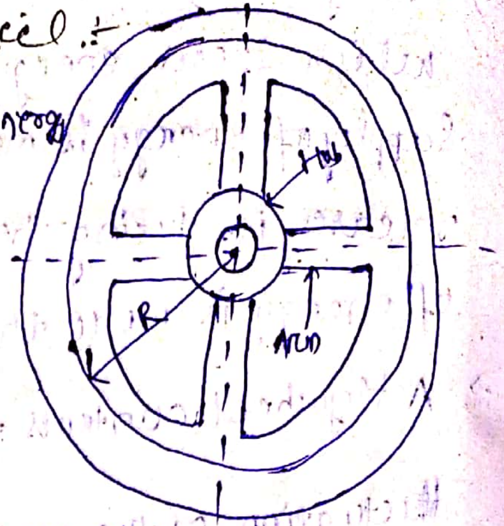
$$C_s = \frac{N_1-N_2}{N} = \frac{2(N_1-N_2)}{N_1+N_2}$$

$$= \frac{\omega_1-\omega_2}{\omega} = \frac{2(\omega_1-\omega_2)}{\omega_1+\omega_2} \quad \text{....(In terms of angular speed)}$$

$$= \frac{v_1-v_2}{v} = \frac{2(v_1-v_2)}{v_1+v_2} \quad \text{...(In terms of linear speed)}$$

Fluctuation of energy :-

Energy stored in flywheel :-



when flywheel absorbs energy it's speed increases & when it gives up energy, it's speed decreases.

m = mass of the flywheel

K = radius of gyration of the flywheel in metres.

I = mass moment of inertia of the flywheel about it's axis of rotation in kg-m² = mk^2

N_1 & N_2 = Max. & min. speeds during the cycle in rpm

ω_1 & ω_2 = max. & min. ang. speeds during the cycle in rad/s.

$\omega = \frac{\Delta\theta}{\Delta t}$

$v = \frac{\Delta s}{\Delta t}$

$\Delta\theta = \frac{\Delta s}{r}$

$v = r\omega$

We know that mean kinetic energy of a flywheel,

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\times m \cdot K^2\omega^2 = \frac{1}{2}I\omega^2$$

As the speed of flywheel changes from ω_1 to ω_2, ($I = mk^2$)

the max. fluctuation of energy,

ΔE = Max. KE − Min. KE

$$= \frac{1}{2}I\omega_1^2 - \frac{1}{2}I\omega_2^2 = \frac{1}{2}I(\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2}I(\omega_1 - \omega_2)(\omega_1 + \omega_2)$$

$$= \frac{1}{2}I\omega C_s(\omega_1 + \omega_2) \quad \because C_s = \frac{\omega_1 - \omega_2}{\omega}$$

$$= I\omega C_s \times \omega = I\omega^2 C_s \quad \left(\because \frac{\omega_1+\omega_2}{2} = \omega\right)$$

$$= mK^2\omega^2 C_s \quad \left(\because \frac{1}{2}mk^2\omega^2 = E\right)$$

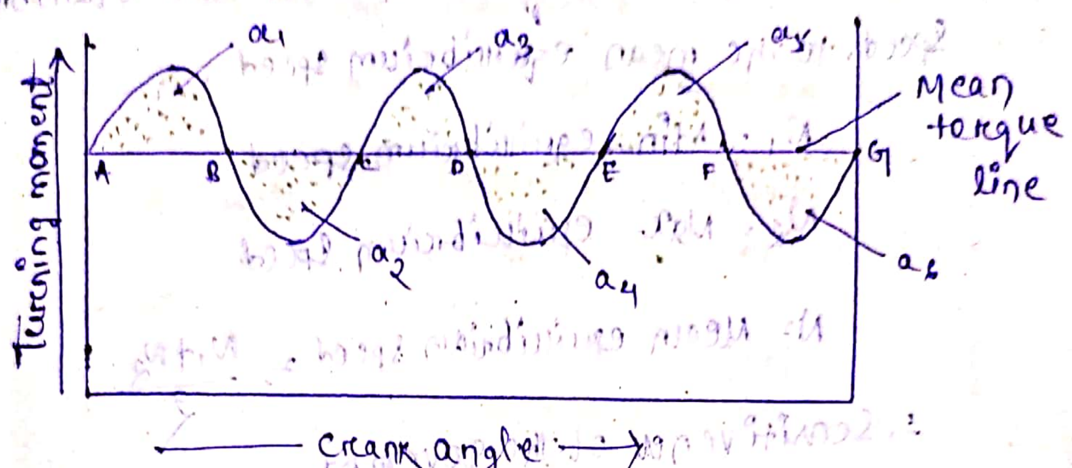
$$= 2EC_s \text{ (in N-m)}$$

Fluctuation of energy :-

→ The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation.

→ The variation of energy above & below the mean resisting torque line are called fluctuation of energy.

Determination of max. fluctuation of energy :-



Let the energy in the flywheel at $A = E$.

Energy at $B = E + a_1$

at $C = E + a_1 - a_2$

at $D = E + a_1 - a_2 + a_3$

at $E = E + a_1 - a_2 + a_3 - a_4$

at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

Let us now suppose that the greatest of these energies is at B & least at E.

∴ Max. energy in flywheel $= E + a_1$

Min. energy in flywheel $= E + a_1 - a_2 + a_3 - a_4$

∴ Max fluctuation of energy,

$$\Delta E = \text{Max. energy} - \text{Min. energy}$$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4)$$

$$= a_2 - a_3 + a_4$$

Sensitiveness of Governor:-

Ratio of the difference betn. the max. & min. equilibrium speeds to the mean equilibrium speed.

N_1 = Min. equilibrium speed

N_2 = Max. equilibrium speed

N = Mean equilibrium speed $= \frac{N_1 + N_2}{2}$

∴ Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

Stability of Governors:-

→ A governor is said to be stable when for every speed within the working range there is a definate configuration i.e. there is only one radius of rotation of the governor balls at which the governor is in equilibrium.

→ For a stable governor, if the equilibrium speed increase the radius of governor ball must also increases.

Isochronous Governor :-

-> A governor is said to be isochronous, when the equilibrium speed is constant (i.e. range of speed is zero) for all radii of rotation of the balls with in the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

Let us consider the case of a porter governor running at speeds N_1 & N_2 rpm. We have discussed that

$$N_1^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1}$$

$$N_2^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_2}$$

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Balancing of machines :- chapter - 5

Static balancing :-

→ A system of rotating masses is said to be in static balance if the combined mass centre of the system lies on the axis of rotation.

→ For static balancing, the vector sum of all the force acting on the rotor is zero.

i.e. $m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2 + \dots = 0$

Where, m = mass
r = radius of rotation
ω = Angular Velocity, rad/sec

$\boxed{\begin{array}{l}\text{Static balancing is to balance only centrifugal force} \\ \text{Dynamic balancing is \sout{to bala} moment equal to zero}\end{array}}$

Dynamic balancing :-

→ Dynamic balancing is the way of balancing ~~machines~~ machines by rotating parts.

→ For dynamic balancing, the algebric sum of the moments ~~also~~ about any point in the plane must be zero.

Causes of unbalance :-

→ Faulty design or manufacture of shaft or rotor (bent shaft) etc.

→ Non homogenity of materials.

→ Faulty mounting of parts causes eccentricity.

→ Mis allignment of bearings.

→ Plastic deformation of certain parts.

→ Wear foundation, loose fitting etc.

→ Thermal gradient, cavitation hammering etc.

→ Non symmetry of the parts.

→ unbalanced centrifugal force in the system.

→ External excitation applied on the system.

~~Effects~~ of unbalance :-

→ Vibration in the rotating machinary is the main effect of unbalance

→ un even wear & tear of rotating parts

→ Quick damage of bearing.

→ (অকাল হওয়া) premature failure of the rotating parts.

→ ~~A.F.~~ Abnormal sound (noise), high friction etc. in the rotating

Q. what's the necessity of balancing of rotating mass

→ Rotating masses on high speed engines or, machines need to be a balanced as far as possible in order to avoid dynamic force to be imparted on them which will cause increase in the load in bearings and various stresses on their membres.

→ It is neccessary for avoiding unpleasent & even dangerious vibrations.

Difference betn static & dynamic balancing.

Static balancing	Dynamic balancing.
→ Static balancing would refers to balancing in a single plane.	→ Dynamic balancing would refers to balancing in more than one plane.
→ It is also known as primary balancing.	→ It is also known as secondary balancing.
→ It is a balance of forces due to action of gravity.	→ It is a balance due to action of inertia forces.
→ Rotation of flywheel, grinding wheels, car wheels are treated as static balancing problems.	→ Rotation of shaft of turbo-generator is a case of dynamic balancing problems.
→ It occurs when the CG of an object is on the axis of rotation.	→ It occurs when the rotation does not produce any resultant centrifugal force or couple. Here the mass of axis is coincidental with the rotational axis.

Chapter-6 :- Vibration of machine parts :-

Amplitude :-

→ It is the max. displacement of a body from its mean position is known as amplitude.

→ The amplitude is always equal to the radius of the circle.

Time period :-

→ It is the time taken for one complete revolution of the particle.

→ $t_p = 2\pi/\omega$

Frequency :-

→ The no of cycles per second is called frequency.

→ It is reciprocal of time period.

→ $n = \omega/2\pi = \frac{1}{t_p}$

→ It is expressed in hertz.

Vibration :-

when an elastic body which is fixed at one end and is displaced at other end from its equilibrium position by the application of external forces, the body starts to move to & fro. Then the body is said to be in vibration.

Types of vibrations :-

i) Free/natural vibration :-

→ when no external force acts on a body, after giving it an initial displacement, then the body is said to be under free or natural vibration.

→ The frequency of natural vibration is called natural frequency.

ii) forced vibration:-

→ when the body vibrates under the influence of external force, the the body is said to be under forced vibrations

→ The frequency of the vibrations is that type of applied force is independent of their natural frequency of vibrations.

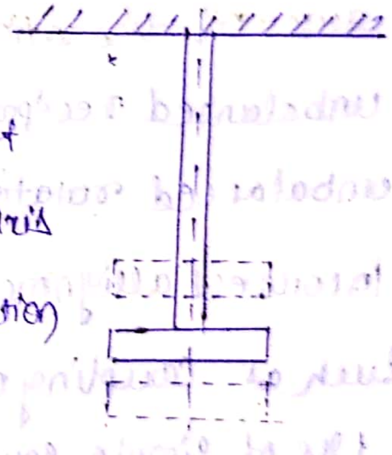
iii) Damped vibration:-

when there is a reduction in amplitude over every cycle of vibration the motion is said to be damped vibration.

Types of free vibrations:-

consider a vibrating body (spring or shaft ised) whose one end is fixed and the other end carrying a heavy disc. the system may execute ~~on~~ the following types of vibrations.

i) Longitudinal vibrations:-



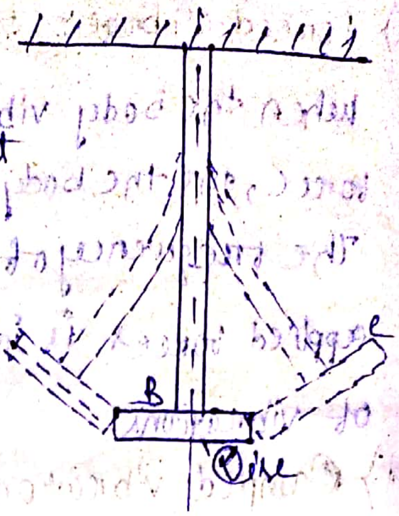
→ when the particle of the shaft or disc moves parallel to the axis of the shaft, then the vibration are known as longitudinal vibrations.

→ In this case tensile & compressive stresses are induced alternately in the shaft.

ii) Transverse vibrations:-

when the particles of the shaft or disc move ⊥r to the axis of the shaft, then the vibration are known as transverse vibrations.

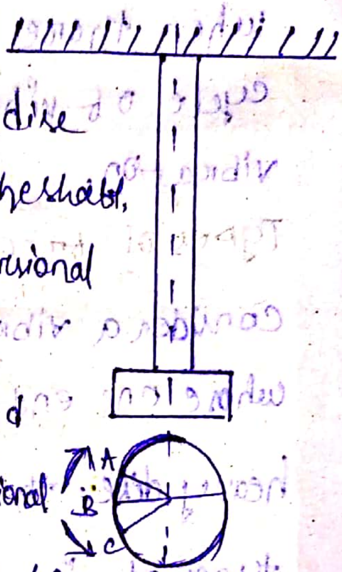
→ In this case bending stress is produced and trough, crest is produced due to vibrations.



(iii) Torsional vibrations

→ When the particle of the shaft or disc move in a circle about the axis of the shaft, then the vibrations are known as torsional vibrations.

→ In this case, the shaft is twisted and untwisted alternately and the torsional shear stresses are induced in the shaft.



Causes of vibrations:-

→ unbalanced reciprocating machine parts.

→ unbalanced rotating machine parts.

→ Incorrect allignment of the transmission elements such as coupling etc.

→ Use of simple spur gears for power transmission.

→ Worn-out teeth of the gears for power transmission.

→ Loose transmission of belts & chain.

→ Loose fastenings of the moving parts.

Remadies of vibration :-

Although it is impossible to eliminate the vibration, yet these can be reduced by adopting various remedies, some of the remedies are listed below.

→ Partial balancing of reciprocating masses.

→ Balancing of rotating machine parts.

→ using helical gears instead of spur gears.

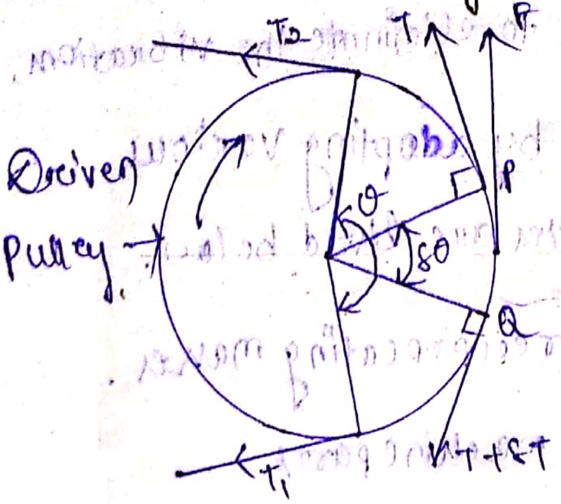
→ proper tightening & locking of fastening & periodically ensuring it again.

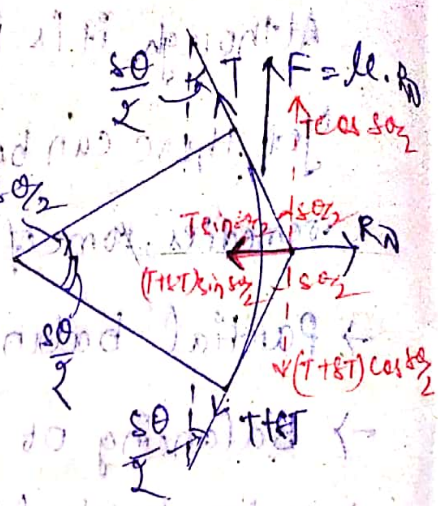
→ correcting the mis-allignment of rotating components & checking it from time to time.

→ Timely replacement of worn out moving parts, slides & bearings with excessive clearance.

Belt rope & chaindrives

Ratio of Driving Tensions for flat belt drive :-





Consider a driven pulley rotating in the clockwise direction.

Let, T_1 = Tension in the tight side.

T_2 = Tension in the slack side.

θ = Angle of contact in radians

μ = co-efficient of friction betn the belt & pulley

Consider a small length of the belt PQ, subtending $\delta\theta$ at the centre of the pulley.

The belt PQ is in equilibrium under the following forces.

I. Tension T in the belt at P, (slackside)

II. Tension $(T+\delta T)$ in the belt at Q (Incr. in tens on tight side than on slack side)

III. Normal reaction R_N &

IV. Frictional force, $F = \mu \times R_N$

Tension T & $T+\delta T$ act in direction perpendicular to the radii draw at the end of the element.

The frictional force μR will act tangentially to the pulley rim resisting the sliping of the elementary belt on the pulley.

Now resolving all the forces in horizontally & equating the same

$$R_N = T \sin\frac{\delta\theta}{2} + (T+\delta T)\sin\frac{\delta\theta}{2} \quad -(I)$$

Since the angle $\delta\theta/2$ is very small, therefore putting $\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$ in eq^n (I), we get

$$R_N = T\frac{\delta\theta}{2} + (T+\delta T)\frac{\delta\theta}{2}$$

$$= \frac{T\delta\theta}{2} + \frac{T\delta\theta}{2} + \cancel{\frac{\delta T\,\delta\theta}{2}} \quad \left(\because \text{neglecting } \frac{\delta T\,\delta\theta}{2}\right)$$

$$R_N = T\delta\theta \quad -(II)$$

Now resolving forces vertically,

$$\mu R_N + T\cos\frac{\delta\theta}{2} = (T+\delta T)\cos\frac{\delta\theta}{2}$$

$$\mu R_N + \cancel{T\cos\frac{\delta\theta}{2}} = \cancel{T\cos\frac{\delta\theta}{2}} + \delta T\cos\frac{\delta\theta}{2}$$

$$\mu R_N = \delta T \quad -(IV) \quad \left(\text{Put } \cos\frac{\delta\theta}{2} = 1\right)$$

Put the value of eq^n (II) in eq^n (IV) we get

$$\mu\, T\delta\theta = \delta T$$

$$\cancel{T\delta\theta = \frac{\delta T}{\mu}} \Rightarrow \mu\,\delta\theta = \frac{\delta T}{T}$$

Now integrating both sides.

$$\int \mu\,\delta\theta = \int_{T_2}^{T_1} \frac{\delta T}{T}$$

$$\mu\theta = \log_e\left(\frac{T_1}{T_2}\right)$$

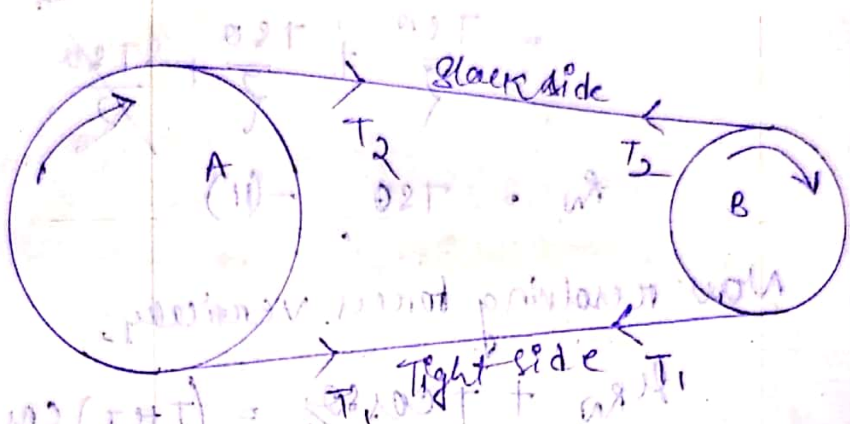
$$\Rightarrow \boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$

Power transmitted by belt drive :-

Let T_1 & T_2 = Tension in the tight & slack side of the belt respectively in newtons.

r_1 & r_2 = radii of the driver & follower respectively

V = velocity of the belt in m/s.



The effective turning /driving force at the circumference of the follower is the difference betn the two tension

i.e $T_1 - T_2$

∴ W.D per second = $(T_1 - T_2) v$ Nm/s

& power transmitted or, $P = (T_1 - T_2) v$ W $\left(\because \frac{N\text{-}m}{s} = W\right)$

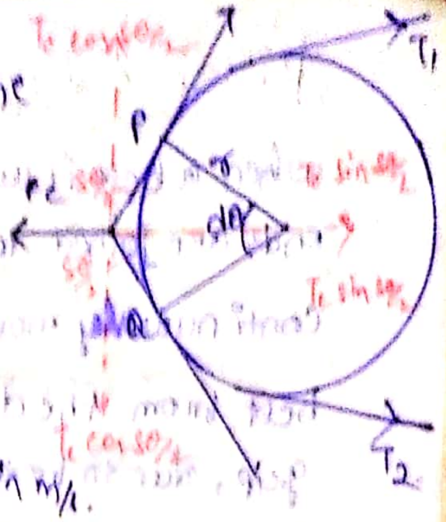
Centrifugal tension :-

→ Since the belt continuously run over the pulleys, there fore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as slack sides.

→ The tension caused by centrifugal force is called centrifugal tension.

→ It is deonated by T_c.

Consider a small portion PQ of the belt at an angle $d\theta$ to the centre of the pulley.



Let, m = Mass of the belt per unit length in kg.

v = linear vel. of the belt in m/s.

r = Radius of the pulley ~~over which the b~~

T_c = centrifugal tension.

Length of the belt, Arc $PQ = d\theta \cdot r$.

mass of the belt $= m.r.d\theta$

centrifugal force acting on the belt PQ,

$$F_c = m\omega^2 r = m.\frac{v^2}{r^2}.r$$

$$= \frac{mv^2}{r}$$

$$= \frac{(mr\,d\theta)v^2}{r}$$

$$\Rightarrow F_c = m\,d\theta\,v^2$$

Equating the forces along horizontally,

$$F_c = T_c\sin\delta\theta/_2 + T_c\sin\delta\theta/_2 = 2T_c\sin d\theta/_2$$

$\delta\theta/_2$ is very small so, let, $\sin d\theta/_2 = d\theta/_2$

$$m\,d\theta\,v^2 = 2T_c\,d\theta/_2$$

$$\Rightarrow T_c = mv^2$$

Initial Tension in the belt (T_0) :-

when a belt is wound round the two pulleys, its two ends are joined together; so that the ~~bat~~ belt may continuously move over the pulleys, since the motion of the belt from the driver & follower is governed by firm grip, due to friction betn the belt & the pulleys.

In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called initial tension.

Let, T_0 = Initial tension in the belt

T_1 = Tension in the tight side of the belt,

T_2 = Tension in the slack side of the belt,

& α = co-efficient of increase of the belt length per unit force

Increase of tension in the tight side = $T_1 - T_0$

~~& " " " in the slack side = $T_0 - T_2$~~

length of the belt in the tight side = $\alpha(T_1 - T_0)$ —(I)

decrease in tension in the slack side = $T_0 - T_2$

length of the belt in the slack side = $\alpha(T_0 - T_2)$ —(II)

Assuming the belt material is perfectly elastic, so that increase in length on the tight side is equal to decrease in length on slack side.

Thus equating eqn (I) & (II) we get,

$$\alpha(T_1 - T_0) = \alpha(T_0 - T_2)$$

$$T_1 - T_0 = T_0 - T_2$$

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$\Rightarrow 2T_0 = T_1 + T_2$

$\Rightarrow T_0 = \frac{T_1 + T_2}{2}$ (Neglecting T_c)

or $T_0 = \frac{T_1 + T_2 + T_c}{2}$ (considering T_c)

Maximum Tension in the belt :-

The tension in the belt (T) is equal to the ~~total~~ tension in the tight side of the belt (T_1)

Let σ = Maximum safe stress in N/mm^2

b = width of the belt in mm, and

t = Thickness of the belt in mm.

We know that,

Max. tension in the belt,

T = Max. stress × cross-sectional area of belt

$T = \sigma \times b \times t$

$T_1 = \sigma . b . t$ ($\because$ when $v < 10 m/s$)

$T_1 + T_c = \sigma . b . t$ ($\because$ when $v > 10 m/s$ then T_c is considered)

Determination of angle of contact :-

When the two pulleys of different diameters are connected by means of an open belt, then the angle of contact or lap (θ) at the smaller pulley must be taken into consideration.

let r_1 = radius of larger pulley
r_2 = radius of smaller pulley
x = Distance betn centre of two pulleys.

$$\sin\alpha = \frac{O_1N}{O_1O_2} = \frac{r_1 - r_2}{x}$$

∴ Angle of contact,

$$\theta^\circ = (180^\circ - 2\alpha)^\circ$$

$180^\circ = \pi$ radian

$1^\circ = \frac{\pi}{180}$ radian

$\theta^\circ = \frac{\pi}{180}(180 - 2\alpha)$ radian (for open belt drive)

$\theta^\circ = \frac{\pi}{180}(180 + 2\alpha)$ rad. (for cross belt drive)

i) $T_{max} = 3T_c$

ii) when max. power occurred, then $v = \sqrt{\frac{T_{max}}{3m}}$

iii) Centrifugal tension, $T_c = mv^2$

iv) $\rho = \frac{m}{V}$; $\Rightarrow m = \rho . V$

$\Rightarrow m = \rho . l . b . t$

Assume leather belt,

$\rho = 1000 \, kg/m^3$

$l = 1m$

$\therefore m = 1000 \, b . t$

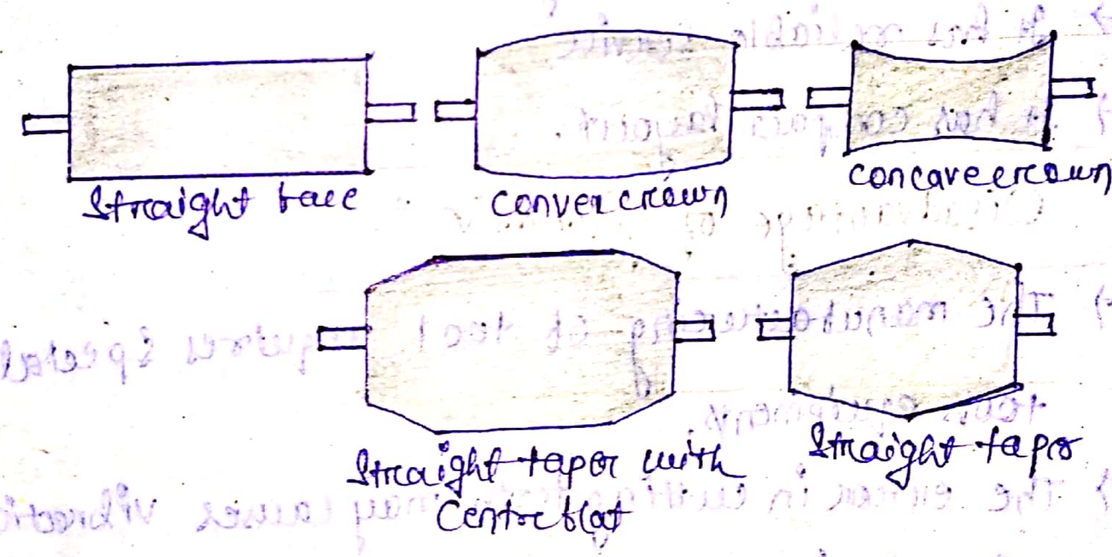
Concept of crowning of pulley :-

→ In a flat belt pulley, the rim surface is given a convex shape by increasing the thickness of a rim at the centre. This increased thickness is called crown & the process is known as crowning of pulley.

It's objective :-

→ In a flat belt drive, if the two shaft are not exactly parallel, there is tendency of belt to come off from the pulley in running condition. The crowning prevents the coming off of the belt from the pulley.

→ The crowning helps to keep the belt near the mid plane of the pulley in running conditions.



Gear drive :-

→ Gear drive is used when centre to centre distance betn driver & driven shaft is very small.

→ It is defined as "toothed wheels, which can transmit power and motion both by one shaft to another by means of successive arrangement of teeth".

→ It is important to note that, both gears, which are ~~engag~~ engaged, always rotate in opposite direction.

[Gear :- When in a frictional wheel with the teeth cut on it, then it is called as gear.]

Advantage of gear drive :-

→ It is used to transmit large power.

→ It has high efficiency.

→ It has reliable service

→ It has compact layout.

Disadvantage of gear drive :-

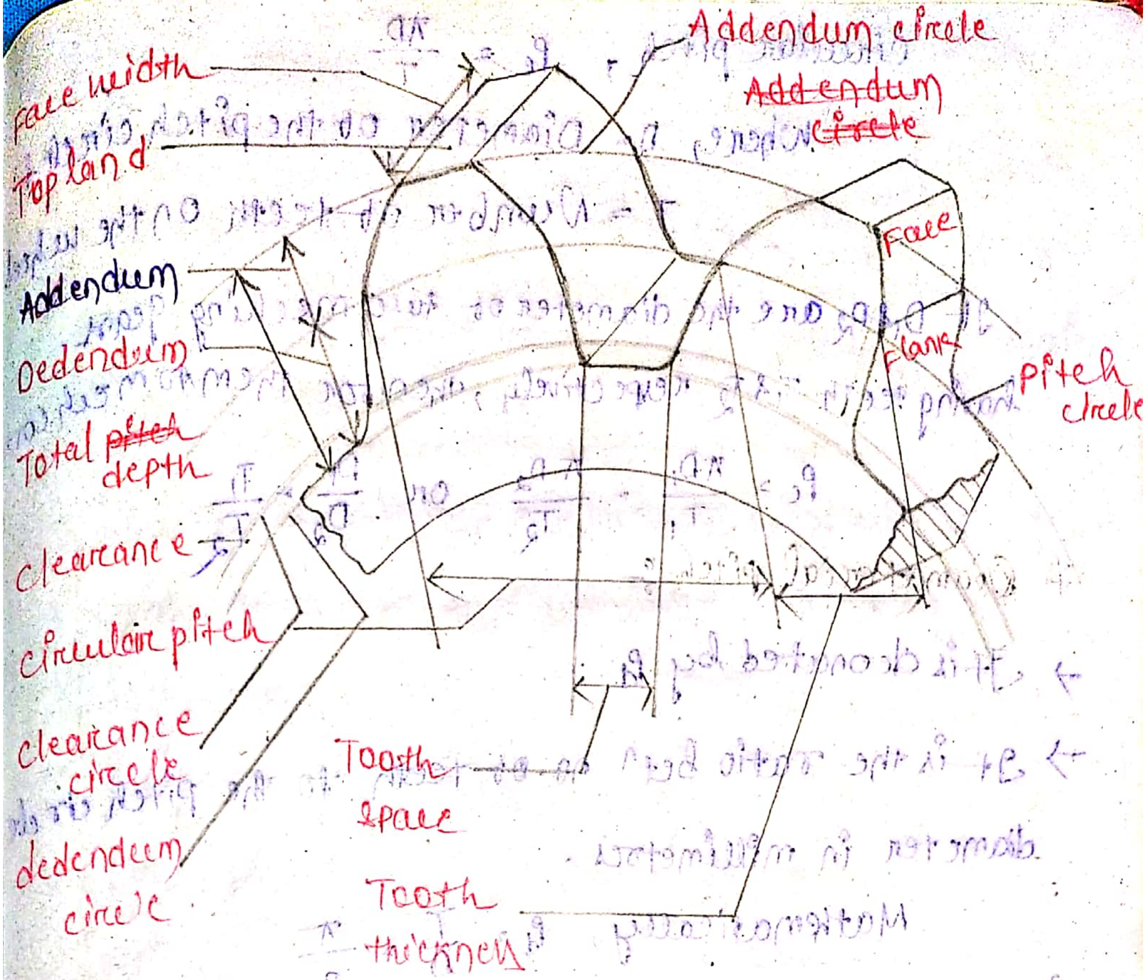
→ The manufacturing of tool requires special tools equipments.

→ The error in cutting teeth may causes vibration & noise during operation.

Terms used in Gears :-

1. pitch circle :- It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. Addendum circle :- It is the circle drawn through the top of the teeth.



3. Dedendum circle :-

It is the circle drawn through the bottom of the teeth.

4. Addendum :-

It is the radial distance of a tooth, from the pitch circle to the top of the tooth.

5. Dedendum :-

It is the radial distance of a tooth, from the pitch circle to the bottom of the tooth.

6. Circular pitch :-

→ It is deonated by P_c.

→ It is the distance measured on the circumference of the pitch circle, from a point of one tooth to the corresponding point on the next tooth.

circular pitch, $P_c = \frac{\pi D}{T}$

where, D = Diameter of the pitch circle &

T = Number of teeth on the wheel.

If D_1 & D_2 are the diameter of two meshing gears having teeth T_1 & T_2 respectively, then for them to mesh corr.

$$P_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

7. Diametral pitch :-

→ It is denoted by P_d.

→ It is the ratio betn no. of teeth to the pitch circle diameter in millimeters.

Mathematically, $P_d = \frac{T}{D} = \frac{\pi}{P_c}$

8. Module :-

→ It is denoted by m

→ It is the ratio of the pitch circle diameter in millimeters to the no. of teeth.

Mathematically, $m = D/T$

Reln betn P_c & m :-

We know that, circular pitch $(P_c) = \frac{\pi D}{T}$

$= \pi \times \frac{D}{T} = \pi \times m$

$\therefore \boxed{P_c = \pi \times m}$

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Gear Trains :-

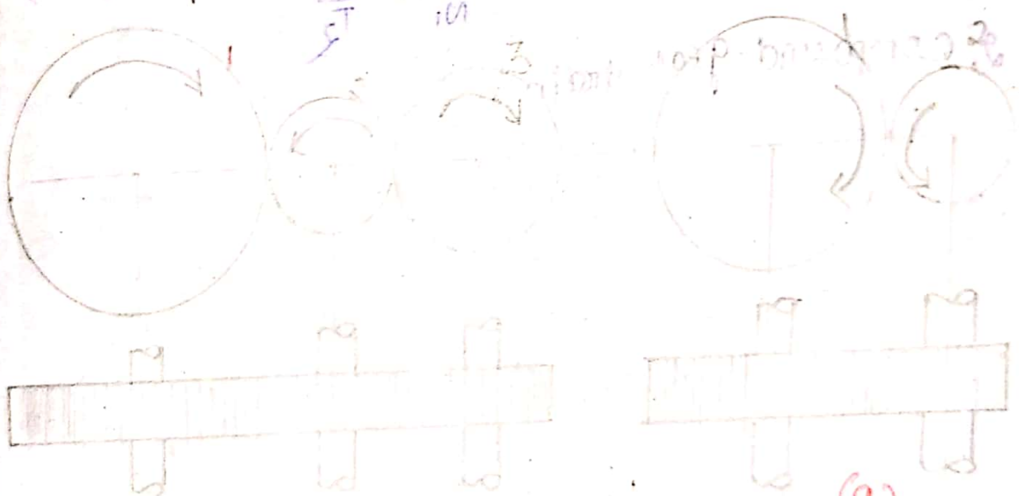
When two or more gears are made to mesh with each other to transmit power from one shaft to another, such a combination is called gear train.

Types of gear train :-

Depending upon the arrangement of wheels, gear trains are classified into following types;

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train

1. Simple gear train :-



(b)

(a)

→ When there is only one gear on each shaft, it is known as simple gear train.

→ In fig-(a), the distance betn the two shafts is small, the two gears 1 & 2 are made to mesh with each other to transmit motion from one shaft to other.

→ The driven/follower gear rotates in opposite dirn of driver gear.

let N_1 = speed of gear -1

N_2 = speed of gear -2

T_1 = No. of teeth on gear -1 &

T_2 = No. of teeth on gear -2.

Since the speed ratio of gear train is the ratio of the speed of the driver to the speed of the follower is equal to their no. of teeth.

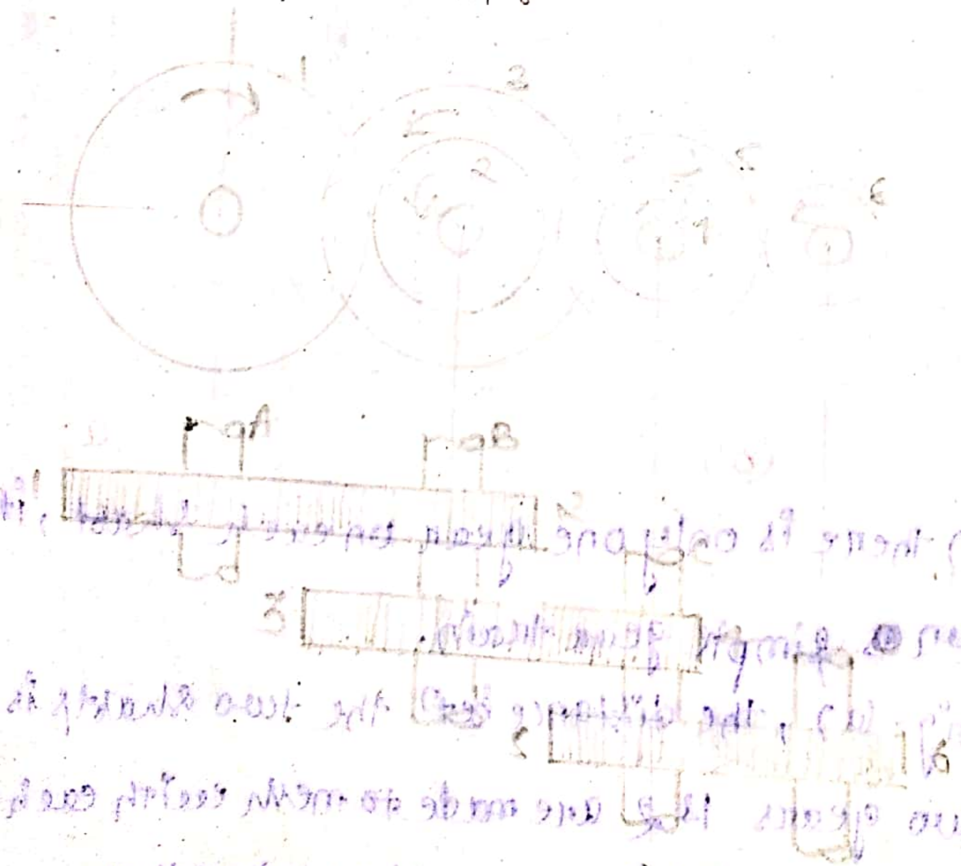
$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

The train value of the gear train is the reciprocal of the speed ratio.

$\left(\because \frac{N_1}{N_2} = \frac{d_2}{d_1} \ \& \ \frac{d_1}{d_2} = \frac{T_1}{T_2} \right)$

i.e. $\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$

2. Compound gear train :-



→ 10. When there are more than one gear on a shaft, it is called a compound train of gear.

→ In a compound geare train, the gear 1 is the driving gear mounded on shaft A, gear 2 & 3 are compound gears which are mounted on shaft B.

→ The gear 4 & 5 are also compound gears which are mounted on shaft C & gear 6 is the driven gear mounted on shaft D.

Let, N_1 = Speed of driving gear -1

T_1 = No. of teeth on driving gear -1

N_2, N_3, N_4, N_5, N_6 = Speed of respective ~~gear~~ gears in rpm

T_2, T_3, T_4, T_5, T_6 = No. of teeth on respective gears

Speed ratio of gear ① & ②

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Similarly, for gear 3 & 4

$$\frac{N_3}{N_4} = \frac{T_4}{T_3}$$

and for gear 5 & 6, speed ratio is

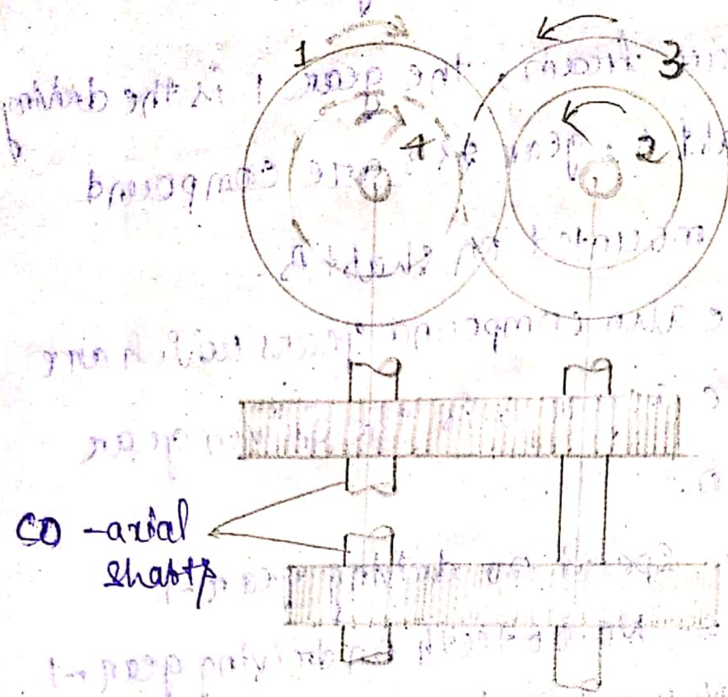
$$\frac{N_5}{N_6} = \frac{T_6}{T_5}$$

Multiply eqn (i), (ii) & (iii), we get,

$$\frac{N_1}{\cancel{N_2}} \times \frac{\cancel{N_3}}{\cancel{N_4}} \times \frac{\cancel{N_5}}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$\Rightarrow \frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

3. Reverted Gear Train :-



→ When the axes of the first gear and the last gear are co-axial, then the gear train is known as reverted gear train.

→ In the following figure gear (1) drives gear (2) in opposite direction.

→ Since gear (2) & (3) are mounted in same shaft, therefore they formed a compound gear & rotates the gear (3) as the same direction of gear (2).

→ The gear (3) drives the gear (4) in the same direction as that of gear 1.

→ ~~So we~~ Thus we see that in reverted gear train, the motion of the first gear & the last gear is like.

Let, T_1 = No. of teeth on gear 1

r_1 = pitch circle radius of gear 1 and

N_1 = speed of gear 1 in rpm.

Similarly,

T_2, T_3, T_4 = No. of teeth on respective gears.

r_2, r_3, r_4 = pitch circle radii of respective gears,

and N_2, N_3, N_4 = speed of respective gears in rpm.

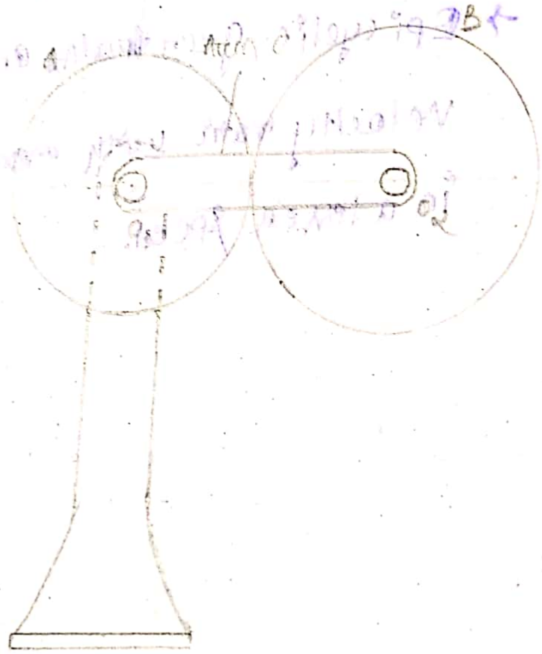
Since the distance bet^n the centres of the shafts of gear 1 & 2 as well as gear 3 & 4 is same.

therefore, $r_1 + r_2 = r_3 + r_4$

Also, $T_1 + T_2 = T_3 + T_4$

$\therefore$ speed ratio $= \dfrac{N_1}{N_4} = \dfrac{T_2 \times T_4}{T_1 \times T_3}$

4. Epicyclic Gear train :-



→ When one gear is fixed & an arm is rotated about the axis of fixed gear & another gear is forced to rotate upon and around the fixed gear, then this type of motion is known as epicyclic.

→ In a gear train, the gears are arranged in such a manner that one or more of their members move upon and around another member is known as epicyclic gear train.

→ In the above figure, gear 'A' & arm have common axis at 'O_1' about which they can rotate.

→ The gear 'B' mesh with gear 'A' & has its axis on the arm at O_2 about which, gear 'B' can rotate.

→ If arm is fixed then the gear train is simple type. i.e. gear 'A' drives gear 'B'.

→ If gear 'A' is fixed & arm is rotated about point 'O', then the gear B is forced to rotate upon & around gear A.

→ Epicyclic gear trains are used for transmitting high velocity ratio with moderate size of gear within a lesser speed.